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The Prime Times

Colgate University

Department of mathematics

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In this issue

Three New Math Faculty

We have three new faces roaming the math departmentÕs halls. *Warren Weckesser* comes to use from his post as an Assistant Professor at the University of Michigan. Prof. Weckesser received his Ph.D. from RPI in 1997. His research includes dynamical systems, mathematical modeling, and mechanical systems. He is also heading the COMAP contest this year. See the announcement below for more details. Next we have *Gaspar Porta* who comes to us from the University of Illinois. His research interest is in operator theory. Our third new member is *Mark Rhodes*, who received his Ph.D. from New Mexico State University. Prof. RhodesÕresearch interest is in commutative algebras.

Please help us in welcoming professors Porta, Rhodes, and Weckesser this year.

Problem solvers wanted for the mathematical contest in modeling: COMAP

Do you enjoy solving challenging problems from the real world? Or, do you enjoy mathematics, or physics, or chemistry, or science in general? Or, do you enjoy mathematics, or physics, or chemistry, or science in general? Or, are you a skilled computer programmer? Or, do you have excellent skills in communication and writing? Or, do you enjoy working with a group of creative problem solvers like yourself?

If you answered yes to any of these questions, you will enjoy the challenge of the *Mathematical Contest in Modeling*. This is a contest organized by the Consortium for Mathematics and its Applications (COMAP), in which teams of up to three students spend an intense weekend solving a real-world problem.

Here are some past problems:

- Asteroid Impact — assess the impact of a 1000m diameter asteroid hitting the earth;
- Maximum Room Capacity --- develop a mathematical model for determining the maximum safe capacity of a public facility;

Problem solvers sought

- Spokes or Solid Disks --- help cyclists determine the conditions under which a solid wheel is more efficient than a spoked wheel.

The contest will take place February 18--22, 2002. We will form teams in October, and run practice problem solving sessions this fall and next semester before the contest.

If you are interested in participating, please contact

Professor Warren Weckesser
314 McGregory Hall
228-7228
wweckesser@mail.colgate.edu

You can also find out more about the contest at the COMAP website: http://www.comap.com

The tetrahedron and the cube: Episode one (Reprised)

By Prof. Lantz

A "(regular) tetrahedron" is a geometric solid, with four faces, all congruent equilateral triangles; so it looks like a pyramid with a three-sided base. I claim that it is impossible to cut up such a tetrahedron into polyhedral pieces and reassemble them into a cube. Unfortunately, the issue is too short to contain the proof. (Sounds a bit like the margin in Fermat's copy of Diophantus.) So I'll try to present it in pieces, like a Dickens novel. In the first chapter we find the measures of the dihedral angles of the figures, i.e., the angles formed by the planes of the faces, measured by their traces on a plane perpendicular to their line of intersection. For the cube, it's clear that all the dihedral angles are right; and for the tetrahedron, they are at least all acute and congruent.

So let's compute some of the distances in the tetrahedron. To keep things straight, let's imagine the tetrahedron's base, the triangle BCD, horizontal and the fourth vertex, A, on top. Let's call the midpoint of side BC the point E, and let's call the foot of the perpendicular from A to the plane of BCD the point F. Then F is the center of triangle BCD. (For an equilateral triangle, all the possible "centers" are identical). And let's agree that all six of the edges of the tetrahedron are one unit long. Then the altitudes AE and DE are the same length (both are one-half the square root of 3), and F is two-thirds of the distance from D to E. Because AFE is a right angle, the cosine of angle AEF is one-third. And because the plane of triangle AFE is perpendicular to edge BC, angle AEF measures the dihedral angle between faces ABC and BCD; so the cosine of that dihedral angle is one-third.

In Episode 2, I'll try to argue that, if the tetrahedron can be cut up and reassembled into a cube, then the inverse cosine of one-third must be a rational number times π. And in Episode 3, I'll show that that's not true, so the proof will be complete.

The tetrahedron and the cube: Episode Two

By Prof. Lantz

In this series, I want to show that it is impossible to cut up a regular tetrahedron into polyhedral pieces (a finite number -- I don't want to consider crushing it) and reassemble them into a cube. In Episode 1 I showed that the dihedral angles in such a tetrahedron measured the inverse cosine of one-third. In this episode, I want to argue that, if this task were possible, then the inverse cosine of one-third would be a rational number times π. To help keep things straight, I'll assume that if one of the cutting planes goes partway through one of these two solids, then it cuts all the way through. As a result, some of the resulting polyhedral pieces may be cut into smaller polyhedral pieces. But it will make it easier to do the summing described in the next paragraph, because it avoids cases in which (1) an edge of one piece may lie in the face of another piece, so that the dihedral angles of the pieces don't entirely surround that edge; or (2) an edge of one of the pieces may be only part of an edge of another piece.

I want to keep track of the sum of the measures of all the dihedral angles of all the pieces that result from cutting up each of these solids. Let's start with the tetrahedron, and look at where each of the edges of the resulting polyhedral pieces are in the tetrahedron, after it is cut but before it is disassembled. Some lie inside of the tetrahedron; the sum of the dihedral angles around such an edge must be 2 π. Some lie in a face of the tetrahedron; the sum of the dihedral angles around such an edge must be π. And some form part or all of an edge of the original tetrahedron; the sum of the dihedral angles around such an edge must be the inverse cosine of one-third. So the sum of all the dihedral angles of all of the pieces is an integer multiple of π plus an integer multiple of the inverse cosine of one-third.

By the same kind of reasoning, if we cut a cube into polyhedral pieces with cutting planes in that way, the sum of the dihedral angles of all the resulting pieces is an integer multiple of π plus an integer multiple of $\pi/4$.

Therefore, if we can cut a tetrahedron into pieces and reassemble them into a cube, the sum of the dihedral angles of all the pieces can be expressed in two ways: as an integer multiple of π plus an integer multiple of the inverse cosine of one-third, and as an integer multiple of π plus an integer multiple of $\pi/4$. Setting these expressions equal and subtracting the first integer multiple of π, we get that an integer multiple of the inverse cosine of one-third is a rational number (with a denominator of 4) times π. So the inverse cosine of one-third is equal to a rational number times π.

In Episode 3, we'll show that the last equation is not true. Therefore, it will follow that the cutting and reassembling is impossible.

WhatÕs in the Hat?

By Prof. Robertson

Here we present a nice little trick created by charles dodgson. The name may not sound familiar to you, but IÕm sure you know him. He wrote fiction under the alias lewis carroll. ThatÕs right, the author of *alice in wonderland* was a mathematician! I bet if you reread *alice in wonderland* you will see many mathematical refereneces and tricks infused into the storyline.

So, hereÕs the situation. We have a hat in which we know there are two marbles. We have only white and black marbles. So, the hat could have either one white and one black marble, both white marbles, or both black marbles. We will now attempt to determine what is in the hat *without looking*!

We will add a black marble to the hat. Note that if the hat contained one white and one black to begin with, then the probability of picking a black marble from the hat after our addition is 2/3. This is the only configuration which gives a probability of 2/3 for picking a black marble after our addition.

So, we now have three marbles in the hat. And we either have three blacks, two whites and one black, or one white and two blacks, each one equally likely. LetÕs calculate the probability of picking a black. If we have three blacks our probability is 1, and this happens 1/3 of the time. If we have one white and two blacks the probability is 2/3, and this happens 1/3 of the time. Finally, if we have two whites and one black the probability is 1/3, and this happens 1/3 of the times. Putting this together gives a probability of picking black of

$$1(1/3)+(2/3)(1/3)+(1/3)(1/3)=2/3.$$

But this means we must have had one white and one black in the hat to begin with. So, by adding a black marble we are able to determine whatÕs in the hat!

Movie and pizza night planned

The math department plans on hosting a movie and pizza night starting next spring. Tentative movies include Good Will Hunting, The Cube, and Π. Tentative pizzas include Plain, Pepperoni, and Mushroom. If you have any ideas for good movies to watch, or good toppings to have on your pizza, please email prof. Robertson at aaron@math.colgate.edu or slip a note in his mailbox . Keep your eye out for announcements.

The math department has a new webpage

The math departmentÕs webpage has gone through yet another renovation. There is a plethora of information there, from alumni comments to job information, and, of course, links to your professors’ webpages and course pages.

If you have a question about a career choice, or just want to browse and see what your professors are up to, check it out. The address is to the left of this box.

WeÕre on the Web!

See us at

http://math.colgate.edu