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The Prime Times

Colgate University Department of Mathematics

Our Best Putnam Team: 20th in USA

The William Lowell Putnam Mathematical Competition is perhaps the world's most difficult exam. Lasting six hours, it has 12 problems worth 10 pts. each. Every December about 3000 undergraduates in the US and Canada, most of them math majors, take the exam. This year, 1900 of the 2900 students taking the test got 0 points, and the highest score was 74 out of 120. Eight Colgate students took the test this year: Bei Shen, Alex Qian, Deni Gintcheva, Joe Converse, Ashwin Lall, Dan Cain, Jon Bloom, and John Zomberg. Each school selects beforehand three students to form the school team; this year the Colgate team was Bei, Alex, and Deni. The team placed 20th of the 431 participating schools, the best Colgate finish ever!

Only the top 10 school names are announced, so it is difficult to determine how Colgate's team placed compared to those of other liberal arts colleges. It looks as though Colgate was either the first or second liberal arts college in the country. No liberal arts college has ever won the team competition, and it is extremely rare for one to finish in the top ten. Colgate also finished ahead of most Ivy League and large public institutions. Note that, unlike athletics, there is no Division structure for big schools and smaller schools, so when the Colgate Putnam team finishes 20th, this is just like the Colgate football team being ranked 20th in Division I, along with Penn State, Florida State, etc.

The best individual performance was by Alex Qian, who ranked 147th of the 3000 students taking the test (51 of the top 200 students came from Harvard or MIT this year). Many of the other Colgate students taking the exam also did quite well: five finished in the top quarter of all students.

The Putnam Eight are already planning for next year's exam. Maybe they will be one of the only liberal arts college teams ever to crack the top ten in the 60 year history of the competition. Anyone interested in joining in the weekly training sessions should see Professor Tucker, Colgate's Putnam coach. Next year the team might have uniforms and laundry service.

Math 312 Helps the Community

The Partnership for Community Development is a non-profit corporation formed by Colgate, the Village of Hamilton, and the Town of Hamilton that seeks to improve the quality of life for low-income residents while respecting the environment.

Prof. Hart and her students in Math 312, Applied Math: Social Sciences, are helping the PCD decide which types of small companies should be recruited to the area.

Prof. Weinberg, of the Sociology and Anthropology Department is the link with PCD.

1st Year Prizes

Nineteen students attempted the 1st Year Prize Exam in Calculus. The first-prize winner ($200) is Alex Qian, and the second-prize winner ($100) is Yibing Cui. The awards will be presented at the honors convocation in May.

COMAP Honorable Mention

One of our teams for the COMAP math modeling competition has received an honorable mention. This competition takes place over an entire weekend and involves solving just one problem. Students may consult any inanimate source, and the team must send a report to the national COMAP office with abstract, bibliography, and computer output.

Bei Shen, Alex Qian, Deni Gintcheva, Joe Converse, Ashwin Lall, and Erica Blomquist served on Colgate's teams.

Students Speak at HRUMC

By the time you read this, the Hudson River Undergraduate Mathematics Conference will be over for this year. Three Colgate students, Jim Smith, Michele Kelly, and Nathan Bailey are scheduled to speak on April 8 at the conference, which is at Vassar this year. The titles of their talks are ÒA Geometric Exploration of the Twin Paradox,Ó ÒAn Awesome Sequence Part I,Ó and ÒAn Awesome Sequence Part II,Órespectively. Others are attending to cheer them on and to hear more talks. The talks are rated by difficulty, and there are several talks going at any given time. This conference is always a fun time. Join us next year!

News of Professors

Did you know? Over the years, at least five of our professors have been nominated for the Phi Eta Sigma Colgate Professor of the Year Award, and two of those professors have won the award.

Prof. Ahlgren gave a major invited hour talk at a conference in Japan in December. He spoke on his research involving modular forms. He also brought Ken Ono, of the University of Wisconsin-Madison, to campus to speak in the Science Colloquium Series as well as to speak to the students in Number Theory. Prof. Ahlgren taught evening classes on number theory to high school students as part of the High School Seminar Program.

Prof. Hart spoke at a research conference in Australia during December on problems in combinatorial algebra arising from topology. She led workshops as part of the Science Day for High School Teachers in March.

Prof. Lantz is Chair of the Department and has had an additional time-consuming appointment to the Promotion and Tenure Committee. He is looking forward to returning to his research on commutative algebras.

Prof. Mastrocola will retire at the end of this semester. He has been a member of the Colgate faculty for 37 years. The department would like to thank him for his service and wish him well.

Prof. Robertson is designing a new course in his specialty, combinatorics, for the fall. He is preparing a research paper on Ramsey Theory for publication, and he has used web-based instruction for his calculus classes. He is our advisor for pre-actuarial students.

Prof. Saracino won the Beta Theta Pi Professor of the Year award in December and a Fraternity and Sorority Award for Excellence in Teaching last spring. The third of a series of his research papers on the model theory of wreath products has just appeared in the Journal of Combinatorial Theory. Prof. Saracino enjoys solving problems that are posed in the American Mathematical Monthly. One of his solutions was recently published there. He has also submitted solutions that he and his students have discovered together. He is one of our Phi Eta Sigma Professor of the Year winners, as mentioned above.

Prof. Schult is spending his sabbatical using mathematical models to study smoldering. With funds from Colgate and from his NSF grant, he will support three students this summer who will help him with his research.

Prof. Strand is preparing a first year seminar on precise reasoning for the fall. He is one of the past winners of the Phi Eta Sigma Professor of the Year Awards mentioned above.

Prof. Tucker has spoken recently at several conferences on topological graph theory, including a conference in Slovenia. He is one of the 10 writers of problems for an NCTM project that is designed to get math problems onto subway posters and restaurant placemats (www.figurethis.org). He is president of The Calculus Consortium for Higher Education, which makes grants to projects in mathematics education.

Prof. Valente has just won a distinguished teaching award from the Inter-Fraternity Council. His latest paper, *Investigating the Contextual Dimensions of Flatland*, has been submitted for publication. He is busy teaching his Core 121 course, The Monument Mathematics, and planning for the extended study component, which involves travel to London and Manchester in May as part of the course.

Brain Teaser

Sophomores Michele Kelly and Nathan Bailey, guided by Prof. Valente, are investigating the dynamics of the sequences such as one that begins
1, 11, 21, 1112, 3112, ...
Prof. Valente first heard of the sequence on a BBC Radio 4 quiz show. Can you find the pattern? The students discovered properties of the sequence and are presenting talks at the HRUMC. See above.

Study Abroad for Math Majors

by Prof. Valente

Some of the most distinctive features of Colgate's curriculum are the many study abroad opportunities open to students. While most off-campus experiences are designed around particular departments and programs, there are at least three groups that have interdisciplinary structures and themes that can accommodate Mathematics concentrators. So, if you are asking yourself "Can I study abroad and still take math courses?" the answer is a resounding "Yes you can."

Which Study Groups?

Although there is no program specifically designed for math concentrators, there are three programs that regularly accept our students and offer electives that can count toward your concentration. These study groups are in Wales [coordinated by the Division of Natural Sciences and Mathematics], Manchester [run by the Division of University Studies], and Australia [overseen by the Environmental Studies program]. Each of these is attached to a prestigious foreign university where you have the status of a visiting student for one semester. Each of these programs has students taking two courses that are developed around the study group theme and/or faculty director and two elective courses chosen from those offered by the host university to its own students.

Can't I take math courses on any Colgate program?

The short answer here is no. Even though Math concentrators can and do accept places on other departmental study groups [Economics or English for example], these programs usually have rigid structures that do not allow for students to chose the courses that they will take abroad. So, if you need a math course while abroad to complete the concentration, you should consider either Wales, Manchester, or Australia.

Which program is best for me?

This isn't an easy question, but I can offer some advice. If the academic focus of a particular program and/or director doesn't appeal to you, then consider another program. The first criteria for any selection should be academics. The timing of the program may also influence your choice. The Wales study group traditionally runs in the spring semester, Manchester and Australia in the fall. Given your progress in the concentration, it may be preferable to be away during a particular semester.

What about non-Colgate programs?

Non-Colgate programs are available to all students, but there is a very important difference to bear in mind. Courses taken on Colgate programs carry Colgate credit, while courses completed on non-Colgate programs are considered transfer credits. Also, participating in a non-Colgate study group will require that you apply for an academic leave of absence, which must be done well in advance of the program dates. Given all of this, there are still rewarding opportunities available to math concentrators, most notably the Budapest Semester in Mathematics (see www.stolaf.edu/depts/math/budapest/intro.html).

How do I prepare?

The first thing you'll need to watch for are the application deadlines for the various study groups. Without exception, you will need to plan at least a year in advance. You and your advisor must plan your concentration around an abroad experience. Here it is important to note that there are two courses required for the concentration that must be completed on-campus [MATH 320 & 323], and we strongly recommend that these courses be completed before the end of the third year. Finally, you will need to discuss the elective mathematics courses at the host university with your advisor to ensure an appropriate choice.

But I have more questions!

Because I (Prof. Valente) have many years of experience with study abroad, including both the Wales and Manchester groups, I'll be happy to answer any other questions you might have. More information about various Colgate and non-Colgate programs is also available in the Off-Campus Study Office, 105 McGregory.

Math Club Party?

Will the math club finish the semester with style? We need ideas and people who can help organize a party. On campus or at Prof. Hart's house? Another movie or games night? Let Prof. Hart or Pete Sparks know what you'd like to have happen.

The Tetrahedron and the Cube

Episode 1. By Prof. Lantz

A "(regular) tetrahedron" is a geometric solid, with four faces, all congruent equilateral triangles; so it looks like a pyramid with a three-sided base. I claim that it is impossible to cut up such a tetrahedron into polyhedral pieces and reassemble them into a cube. Unfortunately, the newsletter is too short to contain the proof. (Sounds a bit like the margin in Fermat's copy of Diophantus.) So I'll try to present it in pieces, like a Dickens novel. In the first chapter we find the measures of the dihedral angles of the figures, i.e., the angles formed by the planes of the faces, measured by their traces on a plane perpendicular to their line of intersection. For the cube, it's clear that all the dihedral angles are right; and for the tetrahedron, they are at least all acute and congruent.

So let's compute some of the distances in the tetrahedron. To keep things straight, let's imagine the tetrahedron's base, the triangle BCD, horizontal and the fourth vertex, A, on top. Let's call the midpoint of side BC the point E, and let's call the foot of the perpendicular from A to the plane of BCD the point F. Then F is the center of triangle BCD. (For an equilateral triangle, all the possible "centers" are identical). And let's agree that all six of the edges of the tetrahedron are one unit long. Then the altitudes AE and DE are the same length (both are one-half the square root of 3), and F is two-thirds of the distance from D to E. Because AFE is a right angle, the cosine of angle AEF is one-third. And because the plane of triangle AFE is perpendicular to edge BC, angle AEF measures the dihedral angle between faces ABC and BCD; so the cosine of that dihedral angle is one-third.

In Episode 2, I'll argue that, if the tetrahedron can be cut up and reassembled into a cube, then the inverse cosine of one-third must be a rational number times pi. And in Episode 3, I'll show that that's not true, so the proof will be complete.

Summer Research Experiences

Many of the professors in the mathematics department have led student research projects during the summer. Projects have included: Expansions of natural numbers in the golden ratio base (Kyle Hunter, '98), Dynamics in filtration combustion (Cem Varon, valedictorian, '98), Integer valued polynomials (Aarti Angara, '99), Solving problems from the Monthly (Jon Bloom, '01), Computer trials of combinatorial conjectures (Cem Varon, '98, Matthew Moser, '97, and Mridul Mehta, salutatorian, '97). This summer three students will work with Prof. Schult modeling smoldering.

Career Info

Math majors make great lawyers, actuaries, economists, professors, biostatisticians, and more. Check out the links assembled on the web at math.colgate.edu/careers.html

Jim Smith, who is studying for a Master of Arts in Teaching, is also creating a web page of interviews with math department alumni. Look for it in the fall.

A Favorite Induction Proof

Reprinted from the MAA Liaisons Newsletter (Based on ideas of Art Benjamin, Harvey Mudd College, and Dan Velleman, Amherst College).

A tromino is a plane figure composed of three squares:

The theorem states that if n is any power of 2, then an n x n square grid with any one of the squares removed can be tiled using trominoes. The base case of this result, when n = 2 (the first power of 2), is clear: a 2 x 2 square with one square removed IS a tromino. For the induction step, suppose that the result holds for an n x n grid with one square removed and consider the next case: a 2n x 2n grid, again with one square removed. Partition this 2n x 2n grid into 4 parts, each an n x n grid. One of the four contains the site of the removed square, and by the induction hypothesis, that is tilable with trominoes. Now there remain three untiled n x n grids. Position one tromino at the point where these 3 meet, so that one square of the tromino lies in each of the three untiled grids. That leaves untiled three n x n grids each with one square removed, and these are tilable again by the induction hypothesis. That completes the proof.

Newsletter Staff

Anyone interested in helping with the departmental newsletter next fall should contact Prof. Hart.