Chinese Remainder Problem - The Beginning.

What is it?

These particular kinds of mathematical problem falls in the category of indeterminate analysis. Usually, it appears in the form as such (in modern notation):
$N = m1x + r1$ $N = m2y + r2$ $N = m3z + r3$ ...

Or in modern number theory notation:
$N \equiv r1 \pmod{m1}$ $N \equiv r2 \pmod{m2}$ $N \equiv r3 \pmod{m3}$ ...

Aside: Writing $N \equiv r1 \pmod{m1}$ [this means N is congruent to r1 modulo m1] means that N divided by m1 leaves r1 as the remainder.

The goal here is to find the smallest positive integer satisfying the congruences states above.

Origins.

Now that you know what a Chinese Remainder Problem is, you must be wondering why or what has this particular kind of problem to do with Chinese Mathematical History. The reason why it is called the Chinese Remainder Problem is because the earliest versions of these congruence problems occured in early Chinese mathematical works. The earliest of such works that contains the Chinese Remainder Problem is the Sun Tzu Suan Ching (also known as Sunzi suanjing) written in approximately late third century by Sun Zi. Problem 26 (also known as the problem of Master Sun) in the third volume of the Sun Tzu Suan Ching offers the earliest recorded Chinese Remainder Problem. Problem 26 is as stated below:

"We have a number of things, but we do not know exactly how many. If we count them by threes we have two left over. If we count them by fives we have three left over. If we count them by sevens we have two left over. How many things are there?" (Quoted from Sun Tze Suan Ching).

Sun Zi's solution is as such:
He first determined the 'use numbers' 70, 21 and 15 which are multiples of 5*7, 3*7 and 3*5 respectively. Next, he noted that the sum (2*70)+(3*21)+(2*15) is equals to 233. Thus 233 is one answer. He then casted out a multiple of 3*5*7 as many times as possible. With this, the least answer, 23, is obtained.

In the modern notation, Sun Zi noticed the following properties:

$70 \equiv 1 \pmod{3} \equiv 0 \pmod{5} \equiv 0 \pmod{7}$,

$21 \equiv 1 \pmod{5} \equiv 0 \pmod{3} \equiv 0 \pmod{7}$,

$15 \equiv 1 \pmod{7} \equiv 0 \pmod{3} \equiv 0 \pmod{5}$.

Hence (2*70)+(3*21)+(2*15) = 233 satisfies the disired congruences. Notice that any multiples of 105 is divisible by 3, 5 and 7. Thus, 2*105 is subtracted from 233 to get 23 is the smallest positive value.

Unfortunately, Problem 26 is the only problem that illustrates the Chinese Remainder Theorem in the Sun Tzu Suan Ching. As such, we cannot determine if he had developed a general method to solve such problems.

Go back to the Chinese History Time Line.