

[Before we begin the proof, note that assumption (ii) of the claim we're going to prove can be 'translated' as follows: If the statement P is true for a string of positive integers starting with c and ending with $m-1$, then the statement $P(m)$ is also true.]

Proof: Consider a statement about the natural numbers, $P(n)$, and a fixed positive integer c with the properties (i) and (ii) as given in the exercise.

Just suppose that $P(n)$ is false for some $n \geq c$.

Fix $T = \{ t \mid t \text{ is a natural number; } t \geq c \text{ and } P(t) \text{ is false} \}$. Now, T is not an empty set because it contains n as an element. So, by the Principle of Well-Ordering, T must contain a smallest element; call it m . Note, since m is in T , $m \geq c$ and $P(m)$ is false.

Now $P(k)$ is true for all k with $c \leq k < m$, because m is the smallest element of T . Thus, by (ii), $P(m)$ must be true as well.

Since we have a contradiction, it must be that $P(n)$ is true for all $n \geq c$. This completes the proof.