Group 6

Groupwork: #10.14

(a) Fix G a group with $|G|=6$. Fix $x \in G$ such that $\text{ord}(x)=6$.

Pf: Because there exists an element $x \in G$ with $\text{ord}(x)=6$

$G=\langle x \rangle$, so G is cyclic.

(b) Fix G a non-cyclic group with $|G|=6$.

For all $x \in G$, $\text{ord}(x) \mid 6$; so $\text{ord}(x)=1,2,3$ or 6.

We know there is exactly one element of order 1.

Because G is not cyclic there is no element of order 6.

Just suppose that $G=\{e, x_1, x_2, x_3, x_4, x_5\}$ with

$\text{ord}(x_i)=2$ $\forall i=1 \ldots 5$. So G is abelian by Exercise 3.11.

Fix $H \subseteq G$ s.t. $H=\{e, x_1, x_2, x_1x_2\}$.

H is closed under multiplication

	e	x_1	x_2	x_1x_2
e	e	x_1	x_2	x_1x_2
x_1	x_1	e	x_1x_2	x_2
x_2	x_2	x_1x_2	e	x_1
x_1x_2	x_1x_2	x_2	x_1	e

Since H is finite, $H \leq G$ by Theorem 5.3.

But $|H|=4$ and $4 \nmid 6$ $\times$ of Lagrange's.

and $\exists\, a \in G$ such that $\text{ord}(a)=3$.