11.14) Want to show that $(\mathbb{Q},+)/(\mathbb{Z},+)$ is an infinite group. Consider $\mathbb{Q}/\mathbb{Z} = \{q + \mathbb{Z} \mid q \in \mathbb{Q}\}$.

Want to show that no two cosets are the same.

Look at $\frac{1}{m} - \frac{1}{n} \in \mathbb{Z}$. $\frac{1}{m} - \frac{1}{n} = \frac{n-m}{nm}$. So then $\frac{n-m}{nm} = p \in \mathbb{Z}$. So $n-m = pnm$. We know that $n \mid m$ and $m \mid n$. But this means that $m=n$ will be the only case in which $\frac{1}{m}$ and $\frac{1}{n}$ will determine the same coset. So then $\{\frac{1}{n}\}_{n=1}^{\infty} + \mathbb{Z}$ are infinitely many in $\mathbb{Q}/\mathbb{Z}$.

Now want to show that every element of $(\mathbb{Q},+)/(\mathbb{Z},+)$ has finite order. Fix $\frac{p}{q} \in \mathbb{Q}$.

Look at $\underbrace{(\frac{p}{q} + \mathbb{Z}) + \cdots + (\frac{p}{q} + \mathbb{Z})}_{q \text{ times}} = q(\frac{p}{q} + \mathbb{Z})$. Then

$q(\frac{p}{q} + \mathbb{Z}) = p + \mathbb{Z} = 0 + \mathbb{Z}$ which is the identity. So then every element has finite order.