Group Assignment 3 Heather Clark, Shirley Han, Claudia Melniciuc, Kellen Myers

12.3) Let G be an abelian group, let n be a positive integer, and let $\varphi : G \to G$ be given by $\varphi(x) = x^n$. Show that φ is a homomorphism. Need it be a monomorphism? An epimorphism?

$\varphi(x\,y) = (x\,y)^n = x^n\,y^n = \varphi(x)\,\varphi(y)$
And thus φ is a homomorphism (an automorphism in fact).

φ is not necessarily monomorphic or epimorphic. It can be (take $n = 1$), but it doesn't have to be.
For example, $n = 4$ when $G = (\mathbb{Z}_8, +)$ defines neither an epimorphism nor a monomorphism.
There is no $g \in \mathbb{Z}_8$ such that $\varphi(g) = 4\,g = 3$. The image of G under φ is actually only $\{0, 4\}$.
So φ is not an epimorphism.
Also $\varphi(0) = 0 = \varphi(2)$, so φ is not a monomorphism.