Group 1

13.7 a) Let $G = (\mathbb{C} - \{0\}, \cdot)$ and let U be the subgroup $U = \{x + yi \mid x^2 + y^2 = 1\}$. Use the Fundamental Theorem to show $G/U \cong (\mathbb{R}^+, \cdot)$

(wts $\theta : G \to \mathbb{R}^+$, θ is onto hom, $\ker \theta = U$)

$\theta = a^2 + b^2$

hom: $\theta(g_1 g_2) = \theta(a+bi)(c+di)$ for $a+bi, c+di \in G$

$\theta((ac - bd) + (bc + ad)i) = (ac - bd)^2 + ((bc + ad)i)^2$

$= a^2c^2 + \cancel{-2acbd} + b^2d^2 + b^2c^2 + \cancel{2adbc} + a^2d^2 = a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2$

$\theta(a+bi) \cdot \theta(c+di) = (a^2+b^2)(c^2+d^2) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$

So $\theta((a+bi)(c+di)) = \theta(a+bi)\,\theta(c+di)$

onto: For some $x \in \mathbb{R}$ show that there is a $g \in G$ such that

$\theta(g) = x$ let $a = \sqrt{x}$, $b = 0$ $\theta(\sqrt{x} + 0i) = x$

$\ker \theta = U$ By the definition of U, $\ker \theta = U$