GROUP I

Exercise 13.7 (b)

DEFINE ϕ such that

$$\phi(a+bi) = \frac{a}{\sqrt{a^2+b^2}} + \frac{b}{\sqrt{a^2+b^2}}i$$

(WTS ϕ is hom.)

Prf:

$$\phi(a+bi)\phi(c+di) = \left(\frac{a}{\sqrt{a^2+b^2}} + \frac{b}{\sqrt{a^2+b^2}}i\right)\left(\frac{c}{\sqrt{c^2+d^2}} + \frac{d}{\sqrt{c^2+d^2}}i\right)$$

$$= \frac{ac}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}} + \frac{ad}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}}i + \frac{bc}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}}i + \frac{bd}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}}i^2.$$

$$= \frac{ac}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}} - \frac{bd}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}} + \frac{ad+bc}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}}i$$

$$= \frac{(ac-bd) + (ad+bc)i}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}} = \phi((a+bi)(c+di))$$

so ϕ is a hom.

Now need to show ϕ is onto an $\mathbb{R}^+$ is the ker(ϕ)

Onto:

Prf:

$U = \{x+yi \mid x^2+y^2=1\}$ so and $x+yi \in U$ gets hit by it self. so ϕ is onto,

$\ker(\phi)$: $\mathbb{R}^+$

Prf:

(We wts $\ker(\phi) \subseteq \mathbb{R}^+$ & $\mathbb{R}^+ \subseteq \ker(\phi)$)

- $\phi : \mathbb{C} \to U$

$\phi(a+bi) = 1+0i$

$$= \frac{a}{\sqrt{a^2+b^2}} + \frac{b}{a^2+bi} i$$

NOTE $B = 0$

so $\frac{a}{\sqrt{a^2+0}} = \frac{a}{|a|}$ but $\frac{a}{|a|} = 1$ so a is positive.

so $\ker \phi \subseteq \mathbb{R}^+$

Fix $r \in \mathbb{R}^+$

$\phi(r) = \frac{r}{\sqrt{r^2+0}} + 0 = \frac{r}{|r|} = 1$ since r is positive

so $\phi(r) \in \ker(\phi)$ for any $r \in \mathbb{R}^+$. So $\mathbb{R}^+ \subseteq \ker \phi$

so $\ker \phi \subseteq \mathbb{R}^+$ & $\mathbb{R}^+ \subseteq \ker \phi$ so $\mathbb{R}^+ = \ker(\phi)$.