14.9) Given that $G, H, K$ are finite abelian groups and $G \times H \cong G \times K$. Then, using thm 14.2, $G$ is determined by $p_1 < p_2 < \ldots < p_m$ attached with $\Pi_1, \Pi_2, \ldots, \Pi_m$ where $\Pi_i = \{0\}$ if $p_i \nmid |G|$. Then $H$ is determined by $p_1 < p_2 < \ldots < p_m$ attached with $\Pi_1', \Pi_2', \ldots, \Pi_m'$ where $\Pi_j = \{0\}$ if $p_j \nmid |H|$. Similarly for $K$.

Then $G \times H$ is determined by $p_1 < p_2 < \ldots < p_m$ attached with $\Pi_1, \Pi_2, \ldots, \Pi_m$ where $\Pi_i$ is the set of all elements in $\Pi_i$ and all elements in $\Pi_i'$.

Similarly for $G \times K$.

So, by thm 14.3, the invariants for $G \times H$ are the same as those for $G \times K$, and because each are unique and determined in the way shown above, then the invariants of $H$ equal the invariants of $K$, so, by thm 14.3, $H \cong K$.

* Note: Let $p_1 < p_2 < \ldots < p_m$ be all of the primes that make up $|G|$, $|H|$, and $|K|$. 

Group 5
Joe Kurstin
Nii Morton