Group 6

17.25

Let R be a Ring. Fix I and J ideals of R.

ⓐ Show $I \cap J$ is an ideal.

Pf: We know that $(I \cap J, +)$ is a subgroup of $(R, +)$ since $(I, +)$ & $(R, +)$ are subgroups of $(R, +)$ and by Thm 5.4.

Fix $x \in I \cap J$ and $r \in R$. So $x \in I$ and $\in J$. Since I & J are ideals, $xr \in I$ and $\in J$. So $xr \in I \cap J$. By the same argument, $rx \in I \cap J$. So $I \cap J$ is ideal.

ⓑ Suppose that I and J are prime. Must $I \cap J$ be prime?

Consider $R = \mathbb{Z}$. $I = \langle\langle 2 \rangle\rangle$ & $J = \langle\langle 3 \rangle\rangle$.

I and J are primes. Consider $I \cap J = \langle\langle 6 \rangle\rangle$.

Look at $4 \in I$ and $9 \in J$. $4 \cdot 9 = 36 \in I \cap J$

but $4 \notin I \cap J$ and $9 \notin I \cap J$.

So $I \cap J$ is not necessarily prime.