

1.6

a). $S = \mathbb{Z}$ $a * b = a + b^2$

• $a * b = a + b^2 \in S$ it is a binary operation

• not commutative:

counterex: $a = 1$ $1 * 2 = 1 + 4 = 5$
 $b = 2$ $2 * 1 = 2 + 1 = 3$

so $1 * 2 \neq 2 * 1$

• not associative:

counterex: $a = 1$ $(1 * 2) * 3 = 5 * 3 = 5 + 9 = 14$
 $b = 2$ $1 * (2 * 3) = 1 * 11 = 122$
 $c = 3$

so $(1 * 2) * 3 \neq 1 * (2 * 3)$

b). $S = \mathbb{Z}$ $a * b = a^2 b^3$

• $a * b = a^2 b^3 \in S$ (\forall) a, b it is a binary operation

• not commutative:

counterex: $a = 1$ $1 * 2 = 1 \cdot 8 = 8$
 $b = 2$ $2 * 1 = 4 \cdot 1 = 4$

so $1 * 2 \neq 2 * 1$

• not associative:

counterex: $a = 1$ $(1 * 2) * 3 = 8 * 3 = 64 \cdot 27$
 $b = 2$ $= 4^3 \cdot 27 = (12)^3$

$c = 3$ $1 * (2 * 3) = 1 * (108) = (108)^2$

so $(1 * 2) * 3 \neq 1 * (2 * 3)$