c). $S = \mathbb{Z}$ $\quad a * b = \dfrac{a}{a^2 + b^2}$

not a binary operation

counterex: $a = 0 \quad b = 0 \quad 0 * 0 \notin S$

d). $S = \mathbb{Z}$ $\quad a * b = \dfrac{a^2 + 2ab + b^2}{a+b} = \dfrac{(a+b)^2}{a+b}$

not a binary operation

counterex: $a = 0 \quad b = 0 \quad 0 * 0 \notin S$

e). $S = \mathbb{Z}$ $\quad a * b = a + b - ab$

- $a * b \in S \ (\forall) \ a, b \in S$ it is a binary operation

- commutative:

$a * b = a + b - ab$ so $a * b = b * a$

$b * a = b + a - ba$ $\quad (\forall) \ a, b \in S$

- associative:

$(a * b) * c = (a + b - ab) * c =$

$= a + b - ab + c - (a + b - ab)c$

$= a + b - ab + c - ac - bc + abc$

$a * (b * c) = a * (b + c - bc) =$

$= a + b + c - bc - a(b + c - bc) =$

$= a + b + c - bc - ab - ac + abc$