

$$i). S = M(2, \mathbb{R}) \quad a = \begin{pmatrix} r_1 & r_2 \\ r_3 & r_4 \end{pmatrix} \quad b = \begin{pmatrix} r_5 & r_6 \\ r_7 & r_8 \end{pmatrix}$$

$$a * b = \begin{pmatrix} r_1 + r_5 & r_2 + r_6 \\ r_3 + r_7 & r_4 + r_8 \end{pmatrix}$$

• $a * b \in S \quad (\forall) \quad a, b \in S$ it is a binary operation

• commutative:

$$a * b = \begin{pmatrix} r_1 + r_5 & r_2 + r_6 \\ r_3 + r_7 & r_4 + r_8 \end{pmatrix} = b * a \quad (\forall) \quad a, b \in S$$

• associative: let $c = \begin{pmatrix} r_9 & r_{10} \\ r_{11} & r_{12} \end{pmatrix} \in M(2, \mathbb{R})$

$$\begin{aligned} (a * b) * c &= \begin{pmatrix} (r_1 + r_5) + r_9 & (r_2 + r_6) + r_{10} \\ (r_3 + r_7) + r_{11} & (r_4 + r_8) + r_{12} \end{pmatrix} = \\ &= \begin{pmatrix} r_1 + (r_5 + r_9) & r_2 + (r_6 + r_{10}) \\ r_3 + (r_7 + r_{11}) & r_4 + (r_8 + r_{12}) \end{pmatrix} = a * (b * c) \end{aligned}$$

$$j). S = P(X) \quad A * B = (A \Delta B) \Delta B \quad \text{where } X \neq \emptyset$$

we know that Δ is a binary operation, so
 $A * B \in P(X) \quad (\forall) \quad A, B \in P(X)$

we also know that Δ is associative:

$$A * B = (A \Delta B) \Delta B = A \Delta (B \Delta B) = A$$