Notes on Exercise 2.9 (b)

You want to show: det(A) ≠ 0 if and only if A is invertible.

As part the proof, many people focused attention on matrix

$$B = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}.$$

To even write this matrix assumes that ad – bc ≠ 0. So any reference to B is only valid in the presence of an assumption that det(A) ≠ 0. That is, B can only be used in a proof of the statement: det(A) ≠ 0 implies A is invertible.

To establish the other implication, you need to prove A invertible implies det(A) ≠ 0. Consequently, you can't refer to B as above since this assumes what you're trying to prove. However you do know A has an inverse (even if you don't know what it looks like). Use this fact together with part (a) of 2.9 to deduce det(A) ≠ 0.