

Some thoughts on Exercise 3.14

You're definitely going to want to invoke Theorem 3.7. However, to do so requires some preliminary work.

One challenge is showing that G with $*$ as described has a single element that can serve as a right identity for all elements of G . For a single element, x , finding a right identity, say z , is an easy matter (based on the assumptions on G). But how do you know that z serves as a right identity for all the other elements of G ?

Hint [continued]: Following on with the notation above, for x in G we know that $xz = x$ for some z in G (Why?). Fix y in G . [WTS $yz = y$ in order to conclude z is a right identity for all elements of G].

Now the properties attached to G allow you to 'connect' x to y by right or left multiplication. If you can connect x to y , perhaps the right identity for x will also be a right identity for y ! Try it.

Once you know that G admits a right identity, z , you have to show that each element in G has a right inverse. But this is easy given your assumptions on G .