4.21) Submitted by Group 5.

Fix $x, y \in G$

So $xy, yx \in G$

Case 1: Let $\operatorname{ord}(xy) = n$ and $\operatorname{ord}(yx) = m$

Then $(xy)^n = e$. So $(xy)(xy)\dots(xy) = e$

Then left multiplying by y and right multiplying by y^{-1} gives $y(xy)(xy)\dots(xy)(y^{-1}) = yy^{-1} = e$. By associativity, $(yx)(yx)\dots(yx) = e$

So $(yx)^n = e$, which implies $m|n$.

And $e = (yx)^m = (yx)(yx)\dots(yx)$. By left multiplying by x and right multiplying by x^{-1}, we get that $(xy)^m = e$, which implies $n|m$.

So $n = m$

Case 2: Let $\operatorname{ord}(yx) = \infty$. Assume $\operatorname{ord}(xy) = p$

So $e = (xy)^p = (xy)(xy)\dots(xy)$. So by manipulation similar to the above, $(yx)^p = e$. $\oplus$

So $\operatorname{ord}(xy) = \infty$