

Problem 5.14

If $H \leq G$ and $K \leq G$, then $H \cap K \leq G$.

Since $e \in H$ and K , $H \cap K \neq \emptyset$.

Fix $a, b \in H \cap K$. Then $a, b \in H$ and

$a, b \in K$. (WTS $ab \in H \cap K$ and $a^{-1} \in H \cap K$.)

Then $ab \in H$ and $ab \in K$ because

$H \leq G$ and $K \leq G$. Then $ab \in H \cap K$.

Also $a^{-1} \in H$ and $a^{-1} \in K$. So $a^{-1} \in H \cap K$.

By theorem 5.1, $H \cap K \leq G$. \square