

## Problem 5.14

If  $H \leq G$  and  $K \leq G$ , then  $H \cap K \leq G$ .

Since  $e \in H$  and  $K$ ,  $H \cap K \neq \emptyset$ .

Fix  $a, b \in H \cap K$ . Then  $a, b \in H$  and

$a, b \in K$ . (WTS  $ab \in H \cap K$  and  $a^{-1} \in H \cap K$ .)

Then  $ab \in H$  and  $ab \in K$  because

$H \leq G$  and  $K \leq G$ . Then  $ab \in H \cap K$ .

Also  $a^{-1} \in H$  and  $a^{-1} \in K$ . So  $a^{-1} \in H \cap K$ .

By theorem 5.1,  $H \cap K \leq G$ .  $\square$