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Exercise 5.2

Let G be the group of real-valued functions on the real line, under addition of functions. Let H be the set of differentiable functions in G. Show that H is a subgroup of G.

Proof:

Note, $f(x) = x^2 \in H$, so $H \neq \emptyset$.

Let $f, g \in H$. So f' and g' exist.

Thus, $(f + g)' = f' + g'$ so $(f + g) \in H$.

The identity in H is $h(x) = 0$.

So $f^{-1} = -f$, and $(-f)' = -(f')$.

So $-f \in H$.

Therefore by Theorem 5.1, $H \leq G$.