Exercise 8.11

Comments added by KGV: ff denotes f^2 etc.

A) Find $Z(S_3)$.

We know that S_3 can be constructed from two functions, f of order 3 and g of order 2, Further that S_3 = {e, f, ff, g, gf, gff}.

e is trivially contained in $Z(S_3)$.

We know that gf=ffg, so neither g nor f nor f*f can be in the center of S_3.

We see that g(gf)=(gg)f = f and that (gf)*g = ff(gg) = ff, so gf is not in the center.

Finally (gff)g = g(ffg) = ggf = f, and that conversely g(gff) = (gg)ff = ff, so gff is not in the center of S_3. This leaves $Z(S_3)$={e}.

B) Find $Z(D_4)$.

We know that D_4 can be constructed from two functions, f of order 4 and g of order 2, Further that D_4 = (e, f, ff, fff, g, gf, gff, gfff}.

e is trivially contained in $Z(D_4)$.

We know gfff ≠ gf = fffg ≠ fg, so g, f and fff are not contained in $Z(S_3)$.

(gf)(gff) = g(fg)ff = g(gfff)ff = f = g(gf) ≠ (gff)(gf) so neither gf nor gff are in the center of D_4 since they fail to commute with each other.

Finally g(gfff) = fff, while (gfff)g = g(fffg) = g(gf) = f, so gfff is not in the center of D_4.

The finally element of D_4 is (f*f), (ff)*f = fff and f*(ff) = fff, so ff might be in $Z(D_4)$.

g(ff) = (gf)f = (fffg)f = (fff)(gf) = (fff)(fffg) = (ff) g, so ff works with both g and f, but since all the elements of D_4 can be constructed by powers of f and g, ff must be commutative with all elements of D_4, so ff is in $Z(D_4)$.

Thus $Z(D_4)$ = {e, ff}.