8.15 Submitted by Group 5

Fix $G$ an abelian group. Let
$H = \{x \in G \mid x^n = e\}$ for a fixed $n$.
Then $x^n = e$, $y^n = e$.

So $(xy)^n = x^n y^n = e$ (using $G$ is abelian)

so $(xy) \in H$.

Now $x^{-1} \in G$

$e = e^n = (xx^{-1})^n = (x^{-1})^n$

so $x^{-1} \in H$

Thus $H \leq G$ by thm 5.1

$S_3 = \{e, f, f^2, g, fg, f^2g\}$.

Consider $H = \{x \in S_3 \mid x^2 = e\}$.

We know $g, (fg) \in H$.

$$
\begin{aligned}
g(fg) &= (gf)g \\
&= (f^{-1}g)g \\
&= f^2
\end{aligned}
$$

But $(f^2)^2 \neq e$, so $H \not\leq S_3$.