Math 102 / Core 143 — Solutions to Exam II — Spring 2003

1. a) 7/13
   b) 11/13
   c) (7/13)(6/12) + (4/13)(9/12) + (2/13)(11/12)
   d) 1-P(none is donkey) $= 1 - (11/13)^7$
   e) $\frac{7!}{5!\,2!}(4/13)^5(9/13)^2$
   f) This is like picking seven times from a box with 4 ones and 9 zeros. The expected value is $EV = 7(4/13) = 2.15$ with $SE = \sqrt{7}\sqrt{(4/13)(9/13)} = \sqrt{7}(6/13) = 1.22$. So the chance of getting between 4.5 and 5.5 ones is the area under the normal curve between $z = (4.5 - EV)/SE = 1.93$ and $z = (5.5 - EV)/SE = 2.75$. $A(1.93) = 94.5\%$ while $A(2.75) = 99.4\%$ so the chance is $0.5 * (A(2.75) - A(1.93)) = 2.45\%$.

2. (10 points)

   a) Box with 49 tickets, 7 with number 65, 27 with number 62 and 15 with number 60. $n = 25$.

   b) Box with 49 tickets, 7 with number 1, 42 with number 0. $n = 15$.

3. (5 points) Wendy did not use a random sample. She chose to ask her friends, which is not representative of incoming college freshmen nationwide. In fact it seems that her friends all go to very good schools, so there is likely to be a confounding variable–most likely that she was a friend of each of them.

4. (20 points)

   a) $EV_{sum} = 100 * 2.5 = 250$ give or take $SE_{sum} = \sqrt{100} * \sqrt{5/4} = 11$.

   b) $EV_{count} = 100 * 0.25 = 25$ give or take $SE_{count} = \sqrt{100} * \sqrt{3}/4 = 4.3$.

   c) $EV_{avg} = 2.5$ give or take $SE_{avg} = \sqrt{5/4}/\sqrt{100} = 0.11$.

   d) $EV_{\%} = 0.25 = 25\%$ give or take $SE_{\%} = \sqrt{100} * \sqrt{3}/4 = 0.043 = 4.3\%$.

5. (15 points) Each box has one ticket with 1 and three tickets with 0 and $n = 344$.
$EV_{count} = n * AVG_{box} = 0.25 * 344 = 86.0$.
$SE_{count} = \sqrt{n} * SD_{box} = (\sqrt{3}/4) * \sqrt{344} = 8.0$.
There are enough of these people/box combinations that the normal distribution should be fairly accurate. The desired area is between $z_1 = (84-86)/8 = -0.25$ and $z_2 = (94-86)/8 = 1$. The table yields $A(1) = 68\%$ and $A(0.25) = 20\%$ so the percentage of these 528 people should be $(68 + 20)/2\% = 44\%$.

6. (20 points)

   a) $EV_{avg} = \$43,000$ give or take $SE_{avg} = \$8,400/\sqrt{900} = \$280$.

   b) $EV_{\%} = 448/900 = 49.8\%$ give or take $SE_{\%} = \sqrt{(448/900) * (452/900)}/\sqrt{900} = 1.67\%$.

   c) Interval from \$42,440 to \$43,560. (Two SEs from the EV)

   d) Using the correction factor for without replacement,
   $SE_{no\ replacement} = \sqrt{\frac{1799-900}{1799-1}} SE_{avg} = \sqrt{1/2}(\$280) = \$198$.