November 16, 2004

Math 111 C — Exam III

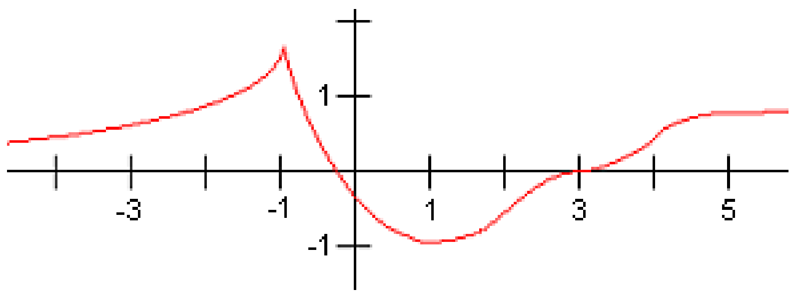
Show all work clearly for partial credit. Do not use the graphing capabilities of your calculator.

1. *(16 points)* Find dy/dx:
   (a) $y = x^2 \ln(1 - x^2)$ (b) $\sqrt{x+y} = 1 + x^2 y$ (answer in terms of x and y)

2. *(10 points)* Find the first and second derivatives of the function $H(t) = \tan(3t)$.

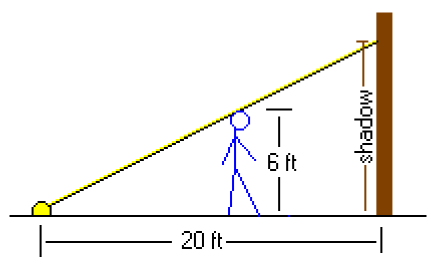
3. *(12 points)* Let $s(t)$ denote the position of a moving point on a number line at time t. If $s(0) = 0$ and the velocity is given by $v(t) = \sin t - \cos t$, find $s(t)$ (in terms of t).

4. *(20 points)* For the graph of $y = f(x)$ below, find (or estimate) the x-values for the following points or intervals:
   (a) where $f'(x)$ doesn't exist;
   (b) where $f'(x) = 0$;
   (c) where $f'(x) \geq 0$;
   (d) where $f''(x) = 0$; and
   (e) where $f''(x) > 0$.

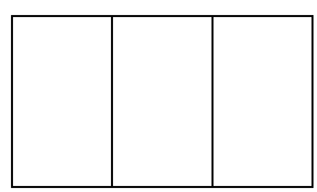


5. *(12 points)* If the edge of a cube is measured as 10 cm with a possible error of 0.2 cm:
   (a) What is (roughly) the possible error when this measurement is used to compute the volume of the cube?
   (b) What is the relative error in the computed volume?

6. *(15 points)* A searchlight is on the ground 20 ft from a wall. A man 6 ft tall walks away from the searchlight toward the wall, casting a shadow on the wall that shrinks as he approaches it. If he is walking 3 ft/sec, how fast is the height of the shadow decreasing when he is 5 ft from the wall?



7. *(15 points)* One hundred twenty feet of barbed wire is to be used to make three adjacent rectangular pens as shown, with one strand of wire along each solid line. What dimensions of the outer rectangle will enclose the greatest total area?



Some possibly useful equations:

$A = \ell w$ $\quad L(x) = f(a) + f'(a)(x - a)$ $\quad$ rel error in $u = du/u$

$dy = \frac{dy}{dx} dx$ $\quad f(b) - f(a) = f'(c)(b - a)$ $\quad V = \frac{4}{3}\pi r^3$

Solutions to Math 111C Exam III

1. (a) $dy/dx = x^2(1/(1-x^2))(-2x) + 2x\ln(1-x^2) = -2x^3/(1-x^2) + 2x\ln(1-x^2)$

   (b) $\frac{1}{2}(x+y)^{-1/2}(1+y') = x^2y' + 2xy$, so $y'(1/(2\sqrt{x+y}) - x^2) = 2xy - 1/(2\sqrt{x+y})$, so

$$\frac{dy}{dx} = y' = \frac{2xy - 1/(2\sqrt{x+y})}{1/(2\sqrt{x+y}) - x^2} = \frac{4xy\sqrt{x+y} - 1}{1 - 2x^2\sqrt{x+y}}$$

2. $H'(t) = 3\sec^2(3t)$, so $H''(t) = 6\sec(3t)(\sec(3t)\tan(3t))3 = 18\sec^2(3t)\tan(3t)$.

3. Because $v = ds/dt$, we have $s(t) = -\cos t - \sin t + C$ for some constant C. Then $0 = s(0) = -\cos 0 - \sin 0 + C = C - 1$, so $C = 1$, and we get $s(t) = 1 - \cos t - \sin t$.

4. By my estimate:

   (a) $x = -1$

   (b) $x = 1$ and $x = 3$

   (c) $x < -1$ and $x \geq 1$

   (d) $x = 2$, $x = 3$ and $x = 4$

   (e) $x < -1$, $-1 < x < 2$, and $3 < x < 4$

5. The volume V of the cube in cm^3 in terms of the edge x in cm is $V = x^3$, so

   (a) $dV = 3x^2\,dx = 3(10)^2(.2) = 60\ \text{cm}^3$.

   (b) $dV/V = 3x^2\,dx\,/\,x^3 = 3dx/x = 3(.2)/10 = .06$

6. Let x denote his distance in ft from the wall, and let s denote the height of the shadow; we are told that $dx/dt = -3$ ft/sec. By similar triangles $6/(20-x) = s/20$, so $s = 120(20-x)^{-1}$. Thus, $ds/dt = -120(20-x)^{-2}(-1)(dx/dt)$, and when $x = 5$ we get $ds/dt = 120(20-5)^{-2}(-3) = -1.6$ ft/sec.

7. Let A denote the area of the outer rectangle in square feet, ℓ be the (horizontal) length in feet, and w be the (vertical) width in feet. Then $A = \ell w$. And because there are 120 ft of wire, we have $2\ell + 4w = 120$, so $\ell = 60 - 2w$, and hence $A = (60-2w)w = 60w - 2w^2$. Now the candidates for the w-value that might give maximum A are the endpoints of the domain — i.e., $w = 0$ and the w-value that makes $\ell = 0$, which is $w = 30$ — and the point(s) where $dA/dw = 0$ — i.e., $60 - 4w = 0$ or $w = 15$. Now if either w or ℓ is 0, then $A = 0$; so $w = 15$ and $\ell = 60 - 2(15) = 30$ must give the maximum value of A (which is $A = 60(15) - 2(15)^2 = 450$, but we weren't asked for that).