December 13, 2004

Math 111 C — Final Exam

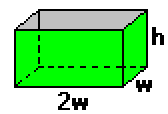
Show all work clearly for partial credit. Do not use the graphing capabilities of your calculator.

1. *(18 points)* Find:

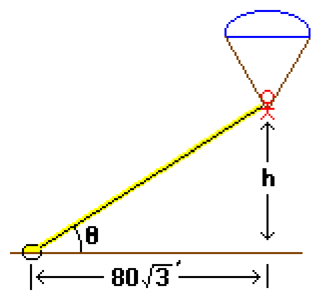
   (a) $\frac{d}{dx}(x\ln(x^2+1))$ (b) $\int(\sin 3x + 2e^x)dx$ (c) $\lim_{x\to-\infty}\frac{x^2}{2x^2+3x-4}$

2. *(12 points)* If $f'(x) = 3x^4 - 8x^3 + 6x^2$, find intervals of concavity and inflection points of the original function f.

3. *(12 points)* A box with an open top is required to have its length twice its width and its volume 288 in^3. What dimensions will give a minimum surface area (not including the top)?



4. *(12 points)* A spotlight on the ground is trained on a parachutist who is descending vertically toward a spot on the ground $80\sqrt{3}$ ft away from the spotlight. When the parachutist is 80 ft above the ground and falling at 12 ft/sec, how fast is the angle between the ground and the beam of the spotlight decreasing, in radians per second?

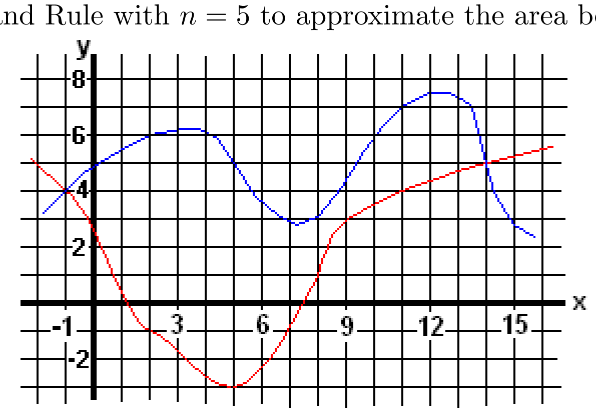


5. *(11 points)* On a planet where the acceleration of gravity is 10 m/sec^2, an astronaut stands at the edge of a cliff and drops a rock (with initial velocity 0). The rock hits the ground at the base of the cliff 3 sec later. How high was the cliff (in meters)?

6. *(18 points)* Evaluate:

   (a) $\int_0^{\ln 2}\frac{e^x}{1+e^x}dx$ (b) $\int_{-1}^{1}\frac{x}{9+x^4}dx$ (c) $\int_0^{\sqrt{3}/2}\frac{dx}{\sqrt{1-x^2}}$

7. *(7 points)* Use the Lefthand Rule with $n = 5$ to approximate the area between the curves:



8. *(5 points)* Display a function $F(x)$ for which $F'(x) = \sqrt{x^3 + 4}$ and $F(2) = 0$.

9. *(5 points)* If the measurement in the side of a square may be off by 2%, by what percent may the computed area of the square be off? Or does the possible percentage error depend on the measurement of the side?

Some possibly useful equations:

$$a^2 + b^2 = c^2 \qquad s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2} \qquad \frac{d}{du}(\arcsin u) = \frac{1}{\sqrt{1-u^2}}$$

Solutions to Math 111C Final Exam

1. (a) $(x/(x^2+1))2x + \ln(x^2+1)$

   (b) $-\frac{1}{3}\cos 3x + 2e^x + C$

   (c)
$$\lim_{x\to-\infty} \frac{x^2}{2x^2+3x-4} = \lim_{x\to-\infty} \frac{1}{2+(3/x)-(4/x^2)} = \frac{1}{2+0-0} = \frac{1}{2}$$

2. $f''(x) = 12x^3 - 24x^2 + 12x = 12x(x-1)^2$, which is 0 when $x = 0, 1$ and never undefined; so we consider the intervals $(-\infty, 0)$, $(0,1)$ and $(1,\infty)$. Now, $f''(-1) < 0$, $f''(1/2) > 0$, and $f''(2) > 0$, so the concavities of f on these intervals are down, up and up respectively. Thus, 0 is a point of inflection; but 1 is not, because the concavity does not change there.

3. Using the notation in the picture, the formula for the volume of a box gives $288 = 2w^2h$, so $h = 144/w^2$. Now adding the surface areas of front and back, two sides and the bottom gives $A = 2(2wh) + 2(wh) + 2w^2 = 6wh + 2w^2 = 864/w + 2w^2$. Thus, $dA/dw = -864w^{-2} + 4w$, which is 0 when $w^3 = 864/4 = 216$, so $w = 6$ in — and then $2w = 12$ and $h = 4$. This gives the minimum area because as w approaches either 0 or ∞, A grows without bound.

4. Because $h/(80\sqrt{3}) = \tan\theta$, we have $(1/(80\sqrt{3}))(dh/dt) = (\sec^2\theta)(d\theta/dt)$. Now when $h = 80$ and $dh/dt = -12$, we have the length of the light beam is $\sqrt{80^2 + (80\sqrt{3})^2} = 160$ (and $\theta = \pi/6$), so $d\theta/dt = (1/(80\sqrt{3}))(-12)/(160/(80/\sqrt{3}))^2 = -9/(80\sqrt{3})$ rad/sec.

5. Because $a(t) = -10$, $v(t) = -10t$ (because $v(0) = 0$) and so $s(t) = -5t^2 + s_0$ where s_0 is the height of the cliff in meters (the height at time 0). Because $s(3) = 0$, we get $s_0 = 5(3)^2 = 45$ m.

6. (a)Using the substitution $u = 1 + e^x$, we get $du = e^x\,dx$ and as x goes from 0 to $\ln 2$, u goes from 2 to 3, so the desired integral becomes
$$\int_2^3 \frac{1}{u}\,du = \ln u|_2^3 = \ln 3 - \ln 2 = \ln(3/2)\ .$$

   (b) It *is* possible to do this problem with the substitution $u = 3x^2$. But the easy way is to realize that $x/(9+x^4)$ is an odd function, being integrated over an interval symmetric about 0, so the integral is 0.

   (c) $\arcsin x|_0^{\sqrt{3}/2} = (\pi/3) - 0 = \pi/3$

7. The interval from -1 to 14 is 15 units long, so one-fifth of it is 3: We find the differences between the two functions at -1, 2, 5, 8 and 11, add and multiply by 3:
$$[(4-4) + (6-(-1)) + (5-(-3)) + (3-1) + (7-4)]3 = 60\ .$$

8. $F(x) = \int_2^x \sqrt{t^3+4}\,dt$

9. If x is the side of the square (measured in some linear unit) and A is the area in square units, then $A = x^2$, so $dA/A = (2x\,dx)/x^2 = 2(dx/x) = 2(2\%) = 4\%$. It doesn't depend on the measurement.