

Math 111 C — Final Exam

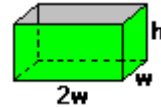
Show all work clearly for partial credit. Do not use the graphing capabilities of your calculator.

1. (18 points) Find:

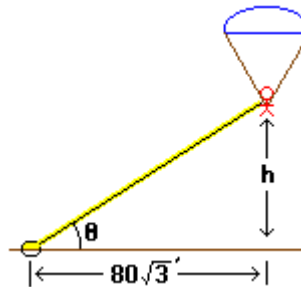
(a) $\frac{d}{dx}(x \ln(x^2 + 1))$ (b) $\int (\sin 3x + 2e^x) dx$ (c) $\lim_{x \rightarrow -\infty} \frac{x^2}{2x^2 + 3x - 4}$

2. (12 points) If $f'(x) = 3x^4 - 8x^3 + 6x^2$, find intervals of concavity and inflection points of the original function f .

3. (12 points) A box with an open top is required to have its length twice its width and its volume 288 in^3 . What dimensions will give a minimum surface area (not including the top)?



4. (12 points) A spotlight on the ground is trained on a parachutist who is descending vertically toward a spot on the ground $80\sqrt{3}$ ft away from the spotlight. When the parachutist is 80 ft above the ground and falling at 12 ft/sec, how fast is the angle between the ground and the beam of the spotlight decreasing, in radians per second?

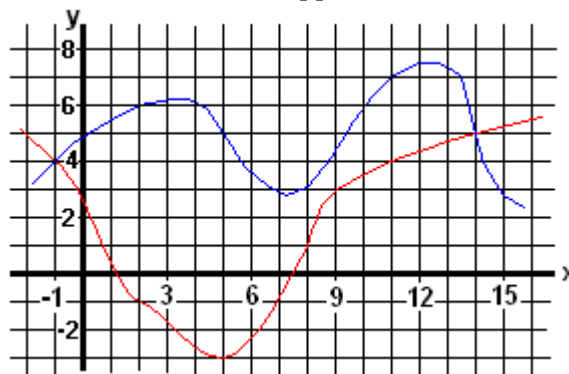


5. (11 points) On a planet where the acceleration of gravity is 10 m/sec^2 , an astronaut stands at the edge of a cliff and drops a rock (with initial velocity 0). The rock hits the ground at the base of the cliff 3 sec later. How high was the cliff (in meters)?

6. (18 points) Evaluate:

(a) $\int_0^{\ln 2} \frac{e^x}{1 + e^x} dx$ (b) $\int_{-1}^1 \frac{x}{9 + x^4} dx$ (c) $\int_0^{\sqrt{3}/2} \frac{dx}{\sqrt{1 - x^2}}$

7. (7 points) Use the Lefthand Rule with $n = 5$ to approximate the area between the curves:



8. (5 points) Display a function $F(x)$ for which $F'(x) = \sqrt{x^3 + 4}$ and $F(2) = 0$.
9. (5 points) If the measurement in the side of a square may be off by 2%, by what percent may the computed area of the square be off? Or does the possible percentage error depend on the measurement of the side?
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Some possibly useful equations:

$$a^2 + b^2 = c^2 \qquad s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2} \qquad \frac{d}{du}(\arcsin u) = \frac{1}{\sqrt{1-u^2}}$$

Solutions to Math 111C Final Exam

1. (a) $(x/(x^2 + 1))2x + \ln(x^2 + 1)$
- (b) $-\frac{1}{3} \cos 3x + 2e^x + C$
- (c)

$$\lim_{x \rightarrow -\infty} \frac{x^2}{2x^2 + 3x - 4} = \lim_{x \rightarrow -\infty} \frac{1}{2 + (3/x) - (4/x^2)} = \frac{1}{2 + 0 - 0} = \frac{1}{2}$$

2. $f''(x) = 12x^3 - 24x^2 + 12x = 12x(x - 1)^2$, which is 0 when $x = 0, 1$ and never undefined; so we consider the intervals $(-\infty, 0)$, $(0, 1)$ and $(1, \infty)$. Now, $f''(-1) < 0$, $f''(1/2) > 0$, and $f''(2) > 0$, so the concavities of f on these intervals are down, up and up respectively. Thus, 0 is a point of inflection; but 1 is not, because the concavity does not change there.
3. Using the notation in the picture, the formula for the volume of a box gives $288 = 2w^2h$, so $h = 144/w^2$. Now adding the surface areas of front and back, two sides and the bottom gives $A = 2(2wh) + 2(wh) + 2w^2 = 6wh + 2w^2 = 864/w + 2w^2$. Thus, $dA/dw = -864w^{-2} + 4w$, which is 0 when $w^3 = 864/4 = 216$, so $w = 6$ in — and then $2w = 12$ and $h = 4$. This gives the minimum area because as w approaches either 0 or ∞ , A grows without bound.
4. Because $h/(80\sqrt{3}) = \tan \theta$, we have $(1/(80\sqrt{3}))(dh/dt) = (\sec^2 \theta)(d\theta/dt)$. Now when $h = 80$ and $dh/dt = -12$, we have the length of the light beam is $\sqrt{80^2 + (80\sqrt{3})^2} = 160$ (and $\theta = \pi/6$), so $d\theta/dt = (1/(80\sqrt{3}))(-12)/(160/(80/\sqrt{3}))^2 = -9/(80\sqrt{3})$ rad/sec.
5. Because $a(t) = -10$, $v(t) = -10t$ (because $v(0) = 0$) and so $s(t) = -5t^2 + s_0$ where s_0 is the height of the cliff in meters (the height at time 0). Because $s(3) = 0$, we get $s_0 = 5(3)^2 = 45$ m.
6. (a) Using the substitution $u = 1 + e^x$, we get $du = e^x dx$ and as x goes from 0 to $\ln 2$, u goes from 2 to 3, so the desired integral becomes

$$\int_2^3 \frac{1}{u} du = \ln u \Big|_2^3 = \ln 3 - \ln 2 = \ln(3/2) .$$

(b) It is possible to do this problem with the substitution $u = 3x^2$. But the easy way is to realize that $x/(9 + x^4)$ is an odd function, being integrated over an interval symmetric about 0, so the integral is 0.

(c) $\arcsin x \Big|_0^{\sqrt{3}/2} = (\pi/3) - 0 = \pi/3$

7. The interval from -1 to 14 is 15 units long, so one-fifth of it is 3: We find the differences between the two functions at $-1, 2, 5, 8$ and 11 , add and multiply by 3:

$$[(4 - 4) + (6 - (-1)) + (5 - (-3)) + (3 - 1) + (7 - 4)]3 = 60 .$$

8. $F(x) = \int_2^x \sqrt{t^3 + 4} dt$

9. If x is the side of the square (measured in some linear unit) and A is the area in square units, then $A = x^2$, so $dA/A = (2x dx)/x^2 = 2(dx/x) = 2(2\%) = 4\%$. It doesn't depend on the measurement.