October 18, 1999

Math 111 E and H — Exam II

Show all work clearly for partial credit. Do not use the graphing capabilities of your calculator.

1. *(36 points)* Find dy/dx:

   (a) $y = x \sin x$ (b) $y = \dfrac{x+5}{x^2+1}$ (c) $y = \tan^2 3x$

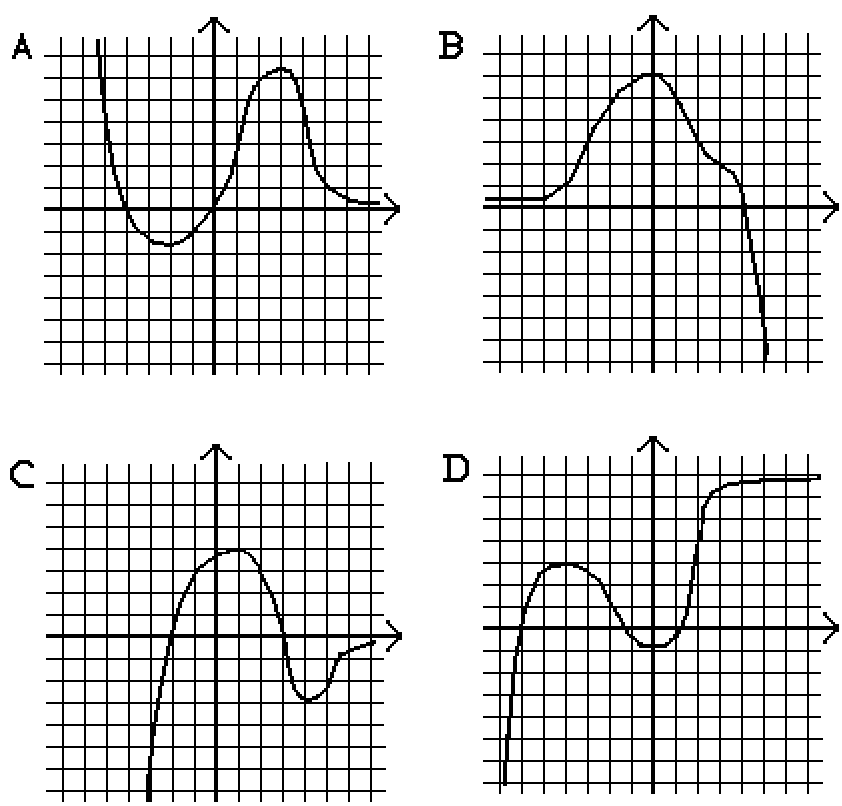
   (d) $y = x^2 - 2^x$ (e) $x^3y - 2e^y = 5$ (in terms of x and y) (f) $y = (x^2+2)^x$

2. *(15 points)* (a) If $y = \sqrt{\ln x}$, what is y'?
   (b) Find the equation of the tangent line to $y = \ln x$ at $x = e^4$.

3. *(15 points)* The distance units on a number line are centimeters (cm). The position of a point on the line at time t sec is given by $s(t) = t^4 - 24t^2 + 8t + 5$. At what (positive) time(s) is (are) the acceleration of the point equal to 0, and what are the position and velocity at this time (these times)? Include units.

4. *(10 points)* If the edge of a cube is growing at 4 inches/min, how quickly is the total surface of the cube increasing when the edge is 20 inches long?

5. *(10 points)* Two of these functions A,B,C,D are the first and second derivatives of another of these, and the remaining function is unrelated to the others. Which of these is f, which is f', which is f”, and which is the unrelated g, and why?



6. *(9 points)* Use the derivative rules for $\sin x$ and $\cos x$ to derive the derivative rule for $\cot x$.

7. *(5 points)* Suppose $y = F(x)$ is a function for which $F'(x) = l/(x^4 + 1)$. Then $F(x)$ is one-to-one, so it has an inverse $F^{-l}(x)$. Express $(F^{-1})'(x)$ in terms of $F^{-1}(x)$. (Hint: From $y = F^{-1}(x)$ we get $x = F(y)$. Differentiate implicitly.)

Some possibly useful equations:

$$y - y_0 = m(x - x_0) \qquad A = \pi r^2 \qquad V = e^3$$

$$A = 6e^2 \qquad A = \frac{1}{2}bh$$

Solutions to Math 111EH — Exam II

1. (a) $y' = x\cos x + \sin x$
   (b) $y' = [(x^2+1) - (x+5)2x]/(x^2+1)^2 = (-x2 - 10x + 1)/(x^2+1)^2$
   (c) $y' = 2\tan 3x(\sec^2 3x)(3) = 6\tan 3x \sec^2 3x$
   (d) $y' = 2x - 2^x \ln 2$
   (e) Because $x^3y' + 3x^2y - 2e^y y' = 0$, we have $(x^3 - 2e^y)y' = -3x^2y$, so $y' = -3x^2y/(x^3 - 2e^y)$.
   (f) Because $\ln y = x\ln(x^2+2)$, we have

$$\frac{1}{y}y' = x\frac{1}{x^2+2}(2x) + \ln(x^2+2) \ , \qquad \text{so} \qquad y' = (x^2+2)^x\left[\frac{2x^2}{x^2+2} + \ln(x^2+2)\right] \ .$$

2. (a) $y' = \frac{1}{2}(\ln x)^{-1/2}\frac{1}{x} = 1/(2x\sqrt{\ln x})$
   (b) When $x = e^4$, $y = \sqrt{\ln e^4} = \sqrt{4} = 2$, and $y' = 1/(2(e^4)(2)) = 1/(4e^4)$, so the desired tangent line is

$$y - 2 = \frac{1}{4e^4}(x - e^4) \ .$$

3. $v(t) = s'(t) = 4t^3 - 48t + 8$, so $a(t) = v'(t) = 12t^2 - 48$, which is zero when $t = 2$ sec (or -2, but the problem says "positive" times), and then $s(2) = 16 - 24(4) + 16 + 5 = -59$ cm, and $v(2) = 4(8) - 48(2) + 8 = -56$ cm/sec.

4. If e denotes the edge of the cube in inches (here e does not mean the base of the natural logarithms) and A is the total surface area in square inches, then we are told that $de/dt = 4$ and we are asked for dA/dt when $x = 20$. Now we have $A = 6e^2$, so $dA/dt = 12e(de/dt)$; so at the instant in question we have $dA/dt = 12(20)(4) = 960$ in^2/min.

5. Function D has zero slopes at -4 and 0, where A has zero values, so a first guess is that A is the derivative of D. This is borne out by the fact that the slopes of D to the left are very positive and to the right are nearly 0 (but positive), and the values of A satisfy these conditions. The slopes and values of B to the left are both nearly 0, which is true of none of the other three; so B is the unrelated function g — it is not the derivative of any of the others, nor are any of the others the derivative of it. Now C has zero values at -2 and 3, which is where A has zero slopes, so we guess that C is the derivative of A. Again, this is borne out by the fact that A has negative slopes to the left and nearly zero (but negative) slopes to the right, as do the values of C. So we conclude that D is the original function f, A is f', and C is f''.

6. $D(\cot x) = D(\cos x/\sin x) = [(\sin x)(-\sin x) - (\cos x)(\cos x)]/\sin^2 x = -(\sin^2 x + \cos 2x)/\sin^2 x = -1/\sin^2 x = -\csc^2 x$

7. 1. From $y = F^{-1}(x)$ we get $x = F(y)$, so

$$1 = F'(y)y' = \frac{1}{y^4+1}y' \ , \qquad \text{so} \qquad (F^{-1})'(x) = y' = y^4 + 1 = (F^{-1}(x))^4 + 1 \ .$$