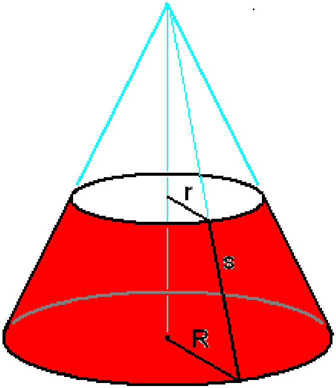
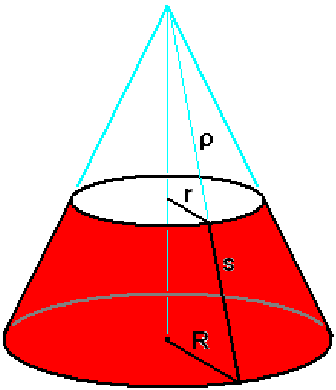
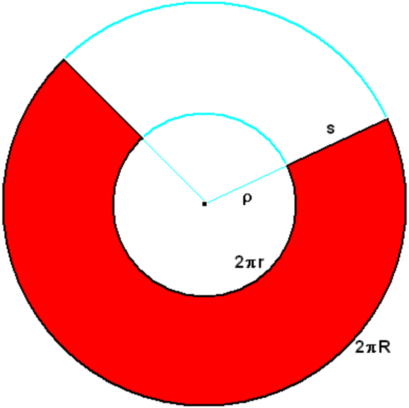
A *frustum of a cone* is the slice of a right circular cone between two planes that are perpendicular to the axis of symmetry of the cone. Its *slant height* (s in the diagram) is the length of a segment from the edge of the top to the edge of the bottom, perpendicular to both. We'll denote the radii of the top and bottom circles by r and R respectively,



Our objective is to express the area of the slanted region of the cone (in red in the diagram) in terms of s and the average of r and R — which is just the radius of a slice halfway between the top and bottom. It is clear that if $r = R$, i.e., if the frustum is a cylinder, then $r = R = (R+r)/2$ and the desired area is $2\pi s(R+r)/2$. We want to show that this formula is valid in general. So we may assume $R > r$. Denote the slant distance down the (removed) part of the cone from the point to the top of the frustum by ρ.



If we cut the surface straight down the side, we can lay it out flat, and it becomes part of the annulus of a circle, with inner radius ρ and outer radius $\rho + s$. The area we want (call it A) is the same fraction of the whole annulus as the circumference of the top of the frustum is of the circumference of the inner circle, or as the circumference of the bottom of the frustum is of the circumference of the outer circle:



$$\frac{A}{\pi(\rho+s)^2 - \pi\rho^2} = \frac{2\pi r}{2\pi\rho} = \frac{2\pi R}{2\pi(\rho+s)}\,.$$

We can solve the last equality for ρ:

$$\frac{2\pi r}{2\pi\rho} = \frac{2\pi R}{2\pi(\rho+s)} \qquad \Longrightarrow \qquad \rho = s\frac{r}{R-r} \ ,$$

and the denominator under A can be simplified:

$$\pi(\rho+s)^2 - \pi\rho^2 = \pi(\rho^2 + 2\rho s + s^2 - \rho^2) = \pi s(2\rho + s) \ .$$

So

$$\begin{aligned} A &= (\pi s(2\rho+s))\left(\frac{r}{\rho}\right) = \left(\pi s\left(2s\frac{r}{R-r} + s\right)\right)\left(\frac{r}{s\frac{r}{R-r}}\right) \\ &= \left(\pi s^2\frac{2r+(R-r)}{R-r}\right)\left(\frac{R-r}{s}\right) = \pi s(R+r) \\ &= 2\pi s\frac{R+r}{2} \ , \end{aligned}$$

as desired.