The problem:

$$\mathcal{P} = \int \frac{-2x^2+6x+12}{(x^2+4)(x^2+4x+6)}\,dx$$

We need the partial fraction expansion of the integrand. First, note that

$$x^2+4x+6 = (x^2+4x+4)+6-4 = (x+2)^2+2\ ,$$

so we will want the numerator over x^2+4x+6 to be expressed in terms of $x+2$ rather than x. So we write:

$$\frac{-2x^2+6x+12}{(x^2+4)(x^2+4x+6)} = \frac{Ax+B}{x^2+4} + \frac{C(x+2)+D}{x^2+4x+6}$$

Then:

$$\begin{aligned} -2x^2 &+ 6x+12 \\ &= (Ax+B)(x^2+4x+6) + (C(x+2)+D)(x^2+4) \\ &= (A+C)x^3 + (4A+B+2C+D)x^2 \\ &\qquad + (6A+4B+4C)x + (6B+8C+4D) \end{aligned}$$

Therefore, setting the coefficients of each power of x on both sides of the equal sign equal to each other, we get:

$$\left\{\begin{array}{ccccccccc} A & & & + & C & & & = & 0 \\ 4A & + & B & + & 2C & + & D & = & -2 \\ 6A & + & 4B & + & 4C & & & = & 6 \\ & & 6B & + & 8C & + & 4D & = & 12 \end{array}\right\}$$

$$\begin{array}{c} \longrightarrow \\ \\ \boxed{2} - 4\cdot\boxed{1} \\ \boxed{3} - 6\cdot\boxed{1} \end{array} \left\{\begin{array}{ccccccc} A & & + & C & & & = & 0 \\ & B & - & 2C & + & D & = & -2 \\ & 4B & - & 2C & & & = & 6 \\ & 6B & + & 8C & + & 4D & = & 12 \end{array}\right\}$$

$$\begin{array}{c} \longrightarrow \\ \\ \boxed{3} - 4\cdot\boxed{2} \\ \boxed{4} - 6\cdot\boxed{2} \end{array} \left\{\begin{array}{cccccccc} A & & + & C & & & = & 0 \\ & B & - & 2C & + & D & = & -2 \\ & & & 6C & - & 4D & = & 14 \\ & & & 20C & - & 2D & = & 24 \end{array}\right\}$$

$$\begin{array}{c} \longrightarrow \\ \\ \boxed{3}/6 \\ \boxed{4} - 20\cdot\boxed{3} \end{array} \left\{\begin{array}{cccccccc} A & & + & C & & & = & 0 \\ & B & - & 2C & + & D & = & -2 \\ & & & C & - & (2/3)D & = & 7/3 \\ & & & & & (34/3)D & = & -68/3 \end{array}\right\}$$

$$\longrightarrow \qquad \left\{ \begin{array}{l} D = \dfrac{-68/3}{34/3} = -2 \\ C = \dfrac{7}{3} + \dfrac{2}{3}(-2) = 1 \\ B = -2 + 2(1) - (-2) = 2 \\ A = -(1) = -1 \end{array} \right\}$$

Therefore,

$$\begin{aligned} \mathcal{P} &= \int \left(-\frac{x}{x^2+4} + 2\,\frac{1}{x^2+4} + \frac{x+2}{(x+2)^2+2} - 2\,\frac{1}{(x+2)^2+2} \right) dx \\ &= -\frac{1}{2}\ln(x^2+4) + 2 \cdot \frac{1}{2}\tan^{-1}\frac{x}{2} + \frac{1}{2}\ln((x+2)^2+2) - 2 \cdot \frac{1}{\sqrt{2}}\tan^{-1}\frac{x+2}{\sqrt{2}} + C \\ &= \ln\sqrt{\frac{x^2+4x+6}{x^2+4}} + \tan^{-1}\frac{x}{2} - \sqrt{2}\tan^{-1}\frac{x+2}{\sqrt{2}} + C \end{aligned}$$