Tricks for Trigonometric Integrals

$\int \sin^m x \cos^n x \, dx$

m odd: $= \int (1 - \cos^2 x)^{(m-1)/2} \cos^n x \sin x \, dx \qquad u = \cos x$

$= -\int (1 - u^2)^{(m-1)/2} u^n du$

n odd: $= \int \sin^m x (1 - \sin^2 x)^{(n-1)/2} \cos x \, dx \qquad u = \sin x$

$= \int u^m (1 - u^2)^{(n-1)/2} du$

m, n both even: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\int \sec^m x \tan^n x \, dx$

m even: $= \int (\tan^2 x + 1)^{(m-2)/2} \tan^n x \sec^2 x \, dx \qquad u = \tan x$

$= \int (u^2 + 1)^{(m-2)/2} u^n \, du$

n odd: $= \int \sec^{m-1} x (\sec^2 x - 1)^{(n-1)/2} \sec x \tan x \, dx \qquad u = \sec x$

$= \int u^{m-1} (u^2 - 1)^{(n-1)/2} du$

m odd, n even: $= \displaystyle\int \frac{\sin^n x}{\cos^{m+n} x} dx$

$= \displaystyle\int \frac{\sin^n x}{(1 - \sin^2 x)^{(m+n+1)/2}} \cos x \, dx \qquad u = \sin x$

and wait for "partial fractions"

$$\begin{aligned}
\sin(A+B) &= \sin A \cos B + \sin B \cos A \\
\sin(A-B) &= \sin A \cos B - \sin B \cos A \\
\cos(A+B) &= \cos A \cos B - \sin A \sin B \\
\cos(A-B) &= \cos A \cos B + \sin A \sin B
\end{aligned}$$

$$\begin{aligned}
\sin A \cos B &= \frac{1}{2}(\sin(A+B) + \sin(A-B)) \\
\sin A \sin B &= \frac{1}{2}(\cos(A-B) - \cos(A+B)) \\
\cos A \cos B &= \frac{1}{2}(\cos(A-B) + \cos(A+B))
\end{aligned}$$