NAME____________________________________

- No calculator is allowed.
- If you have any questions, please raise your hand and ask. The worst that will happen is that I will say, “I can’t tell you.”
- Do the problems that you find easiest first. Take deep breaths between questions.
- There are 100 points on this exam, and you have 110 minutes.
- I hope you all do well. Good luck!

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Points Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Take three deep breaths.

What information have you been given?

What information do you need?

How can you get from one to the other?

Trigonometric Product Formulas:

\[
\sin(A) \sin(B) = \frac{1}{2} \left[ \cos(A - B) - \cos(A + B) \right]
\]

\[
\cos(A) \cos(B) = \frac{1}{2} \left[ \cos(A - B) + \cos(A + B) \right]
\]

\[
\sin(A) \cos(B) = \frac{1}{2} \left[ \sin(A + B) + \sin(A - B) \right]
\]
1. (13 points) Evaluate the following limits. Show your work.

a) \[ \lim_{x \to 0^+} \frac{1}{e^x - 1} - \frac{1}{x} \]

b) \[ \lim_{x \to \infty} \left( \frac{x - 2}{x} \right)^x \]

c) \[ \lim_{\theta \to 0} \frac{\sin(\theta)}{1 + \cos(\theta)} \]
2. (10 points) Evaluate the integral if it converges, otherwise show it diverges.
\[ \int_2^\infty \frac{dx}{(x-7)^4} \]

3. (10 points) Evaluate the integral if it converges, otherwise show it diverges.
\[ \int_4^\infty \frac{dx}{x\sqrt{x-1}} \]
4. (10 points) Use the Comparison Theorem to determine whether or not the following integral is convergent. (You do not need to evaluate the integral.)
\[ \int_{1}^{\infty} \frac{1}{3x^2 + 2\sin(2x) + 2} \, dx \]

5. (7 points) Find the \((x, y)\) coordinates of all the points of intersection for the curves \(r = \sin(\theta)\) and \(r = \cos(\theta)\).
6. (10 points) Set up BUT DO NOT EVALUATE an integral for the area of the region that is inside both $r = \sin(\theta)$ and $r = 1 - \sin(\theta)$.

7. (10 points) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve $x(t) = 3t^5 + 4t + 5$, $y(t) = 3t^2 + 2t + 3$ at the point $(x, y) = (5, 3)$.
8. (10 points) Consider the cochleoid given by $r = \begin{cases} \frac{\sin(\theta)}{\theta} & \theta \neq 0 \\ A & \theta = 0 \end{cases}$.

a) Find the value for $A$ which makes this function a continuous function.

b) Sketch the resulting curve for $0 \leq \theta \leq 2\pi$. 
9. (10 points) For the following polar curves find the equation for the curve in cartesian coordinates. Then identify the curve (i.e. using English).

   a) \( \tan(\theta) = 5 \)

   b) \( r = \frac{1}{1 + \cos(\theta)} \).

10. (10 points) Find the length of the curve given in polar coordinates as \( r = e^{\theta/2} \), for \( 0 \leq \theta \leq 2\pi \).