Chapter 13, Section 7, Exercise 20

Though I didn't assign this one, let us look at a solution: Writing down the definitions for f_{xy} and f_x and assuming that all the limits behave well, we get:

$$\begin{aligned}
f_{xy}(a,b) &= \lim_{k\to 0} \frac{f_x(a,b+k) - f_x(a,b)}{k} \\
&= \lim_{k\to 0} \frac{\lim_{h\to 0} \frac{f(a+h,b+k) - f(a,b+k)}{h} - \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}}{k} \\
&= \lim_{k\to 0} \frac{\lim_{h\to 0} \frac{f(a+h,b+k) - f(a,b+k) - f(a+h,b) + f(a,b)}{h}}{k} \\
&= \lim_{k\to 0} \lim_{h\to 0} \frac{f(a+h,b+k) - f(a,b+k) - f(a+h,b) + f(a,b)}{hk}
\end{aligned}$$

If we did the same thing with f_{yx} and f_y, we would get almost the same expression:

$$f_{yx}(a,b) = \lim_{h\to 0} \lim_{k\to 0} \frac{f(a+h,b+k) - f(a+h,b) - f(a,b+k) + f(a,b)}{hk}$$

The only difference is the order in which we take the limits. So if the limits behave well with respect to each other, we should get the same result.