Example:

$$\begin{aligned}
\int_0^1 \int_0^z \int_0^{x+z} 6xz\,dy\,dx\,dz &= \int_0^1 \int_0^z [6xzy]_{y=0}^{y=x+z}\,dx\,dz \\
&= \int_0^1 \int_0^z (6x^2z + 6xz^2)\,dx\,dz \\
&= \int_0^1 \left[2x^3z + 3x^2z^2\right]_{x=0}^{x=z}\,dz \\
&= \int_0^1 5z^4\,dz = \left[z^5\right]_0^1 = 1\ .
\end{aligned}$$

Thus, if $6xz$ is the density (mass per unit volume — say, grams per cubic centimeter, where the variables are all measured in centimeters) of a solid as in the graphical example from this section, then the total mass of the solid is 1 (gram, say). Now suppose that we want the y-coordinate of the center of mass of the solid. By the formula in class (or in the text's problem set for Section 15.3), it is

$$\begin{aligned}
\overline{y} &= \frac{1}{1}\int_0^1 \int_0^z \int_0^{x+z} y6xz\,dy\,dx\,dz \\
&= \int_0^1 \int_0^z \left[3xzy^2\right]_{y=0}^{y=x+z}\,dx\,dz \\
&= \int_0^1 \int_0^z (3x^3z + 6x^2z^2 + 3xz^3)\,dx\,dz \\
&= \int_0^1 \left[\frac{3}{4}x^4z + 2x^3z^2 + \frac{3}{2}x^2z^3\right]_{x=0}^{x=z}\,dz \\
&= \int_0^1 \frac{17}{4}z^5\,dz = \left[\frac{17}{24}x^6\right]_0^1 = \frac{17}{24}\ ;
\end{aligned}$$

i.e., the center of mass is $\frac{17}{24}$ cm (if that is the unit of length) on the positive side of the xz-plane.