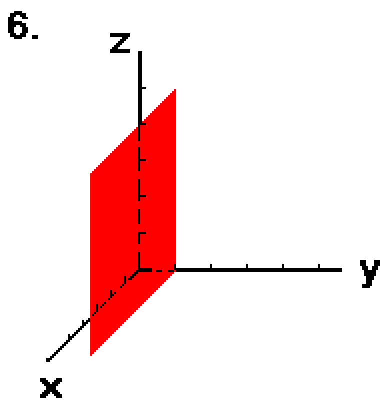
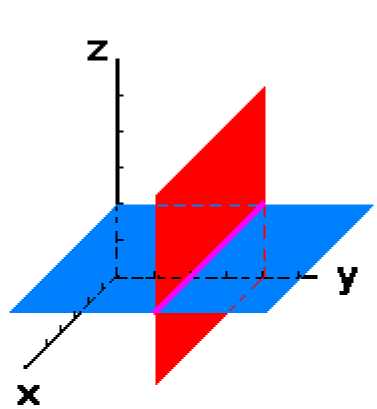
Problems 11.2, Page 13

4. At point (1,-1,1). In front of the yz-plane, to the left of the xz-plane, and above the xy-plane.

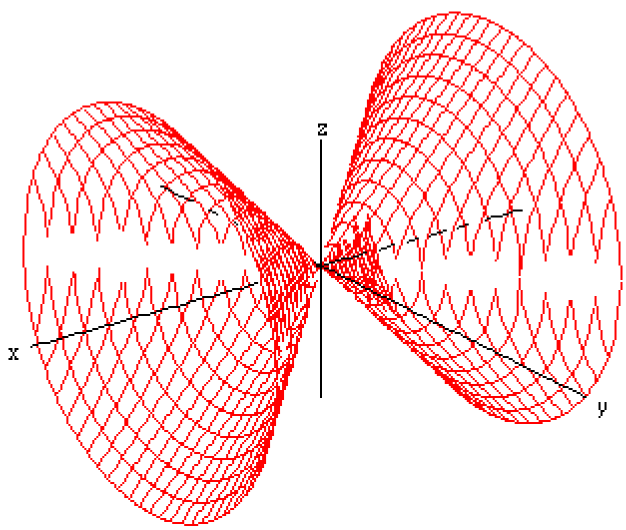




If in Problem 7 we want the points where both (rather than at least one) of the equations are true, we want the violet line of intersection of the red and blue planes, not the union of the planes themselves.

8. The closest point on the y-axis to the point $P = (a, b, c)$ is the point where the plane through P parallel to the xz-plane hits that axis. The equation of that plane is $y = b$, so the point is $Q = (0, b, 0)$, and the desired distance is the distance from P to Q, i.e., $\sqrt{a^2 + c^2}$.

10. Denote a general point by $P = (x, y, z)$. Its distance from the x-axis is $\sqrt{y^2 + z^2}$ (as in Problem 8), and its distance from the yz-plane is $|x|$; so the conditions of the problem can be stated $\sqrt{y^2 + z^2} = |x|$, or equivalently $y^2 + z^2 = x^2$. Fixing various x-values, we can see that slicing this figure parallel to the yz-plane results in circles around the origin, growing larger as the x-value grows in absolute value; and setting $y = 0$ or $z = 0$ shows that the figure hits the xz- and xy-planes in two lines crossing at the origin ($z = \pm x$ and $y = \pm x$, respectively). So the figure consists of two infinite cones, their points meeting at the origin, with axis of rotation the x-axis.



(To use the terminology more correctly, there is only one cone here, infinite in both directions of the x-axis. Each of what we usually call a "cone" is more correctly called a *nappe* of the complete cone.)

11. $P_1P_2 = \sqrt{(3-1)^2 + (2-2)^2 + (1-3)^2} = \sqrt{8}$, $P_1P_3 = \sqrt{(1-1)^2 + (1-2)^2 + (0-3)^2} = \sqrt{10}$, and

$P_2P_3 = \sqrt{(1-3)^2+(1-2)^2+(0-1)^2} = \sqrt{6}$; so P_2 and P_3 are closest together.

19. $\sqrt{(x-1)^2+(y-2)^2+(z-3)^2} = 5$, or equivalently $(x-1)^2+(y-2)^2+(z-3)^2 = 25$.

20. (a) The points in the xy-plane satisfy $z = 0$, so the equation of the desired circle is $(x-1)^2+(y+3)^2+(0-2)^2 = 4$, or equivalently $(x-1)^2+(y+3)^2 = 0$, which is a single point, $(1,-3)$. Similarly, the given sphere meets the xz-plane when the equation $(x-1)^2+(0+3)^2+(z-2)^2 = 4$, or $(x-1)^2+(z-2)^2 = -5$, holds; but there are no such points. Finally, the sphere meets the yz-plane when $(0-1)^2+(y+3)^2+(z-2)^2 = 4$, or $(y+3)^2+(z-2)^2 = 3$, holds, which is a circle with center $(-3,2)$ and radius $\sqrt{3}$ in that plane.

(b) The sphere hits the x-axis when y and z are 0, so the desired points satisfy $(x-1)^2+(0+3)^2+(0-2)^2 = 4$, or $(x-1)^2 = -9$, and there are no such points. We could have seen this by noting that the x-axis lies in the xz-plane, and we saw in (a) that the sphere doesn't meet this plane. By the same token, the sphere doesn't meet the z-axis. It hits the y-axis at points where $(0-1)^2+(y+3)^2+(0-2)^2 = 4$, or $(y+3)^2 = -1$; so the sphere doesn't hit this axis either.