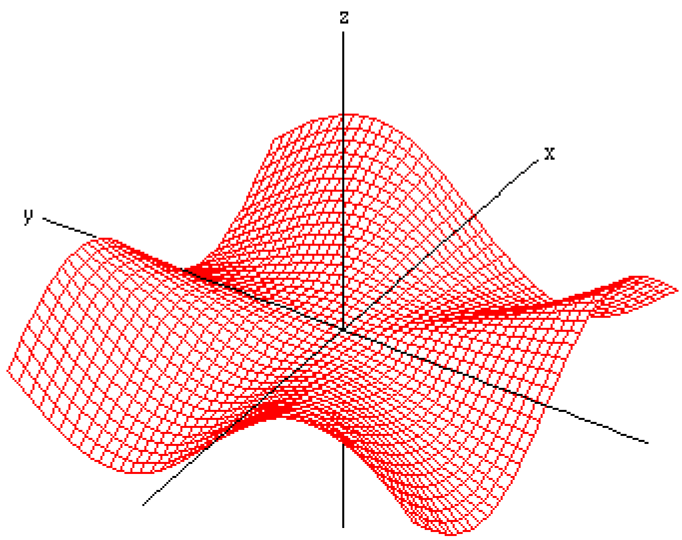
Problems 11.7, Page 55

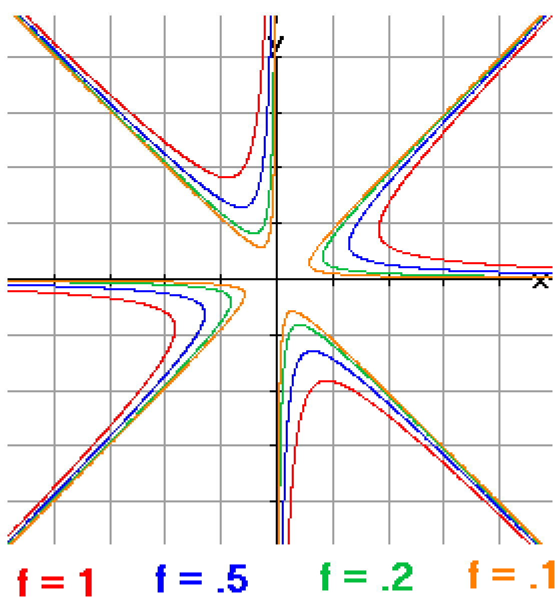
1. The curves $y = kx^2$ all approach $(0,0)$ as $x \to 0$, so if the limit of $f(x,y) = x^2/(x^2+y)$ exists, it must be the same as the limit of $f(x, kx^2)$ for every value of k. But

$$\lim_{x\to 0} \frac{x^2}{x^2 + (kx^2)} = \frac{1}{1+k} \ ,$$

so the limit is different for different values of k; and hence the limit of f as $(x,y) \to (0,0)$ does not exist.

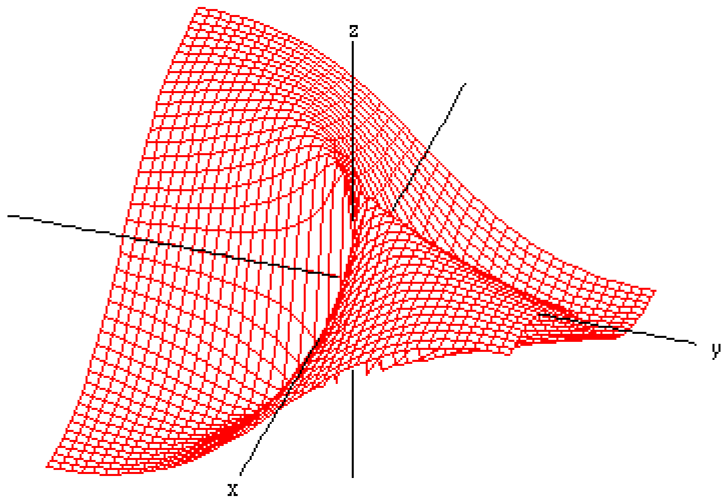
3. (a)





(b) Yes, it appears that in any neighborhood of $(0,0)$, the values of $f(x,y)$ are close to 0, which is $f(0,0)$ by definition.

4. Figure 11.104 in the text is hard to interpret. Here is a better picture:



(a) Because $f(0, y)$ and $f(x, 0)$ are the constant functions 0, they are clearly continuous.
(b) We saw in (a) that the rays in the y-axis, where $x = 0$, have this property. So take the ray $y = kx$ from the origin: On that ray,

$$f(x, kx) = \frac{x(kx)}{x^2 + (kx)^2} = \frac{k}{1 + k^2} \ ,$$

which is a constant (depending on k). Thus, the entire ray lies on the contour $f = k/(1 + k^2)$.
(c) No, because different paths to the origin approach different z-values, even along the straight-line paths as we saw in (b) — different values of k yield different limits.