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1.

$$\vec{k} \times \vec{\jmath} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \vec{\imath} - \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \vec{\jmath} + \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} \vec{k} = -\vec{\imath}$$

2. No: $\vec{\imath} \times \vec{\imath}$ is a vector (the zero vector), and $\vec{\imath} \cdot \vec{\imath}$ is a scalar (1).

3.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \vec{\imath} - \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \vec{\jmath} + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \vec{k} = -\vec{\imath} + \vec{\jmath} + \vec{k}$$

6.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & -3 & 1 \\ 1 & 2 & -1 \end{vmatrix} = \begin{vmatrix} -3 & 1 \\ 2 & -1 \end{vmatrix} \vec{\imath} - \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} \vec{\jmath} + \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} \vec{k} = \vec{\imath} + 3\vec{\jmath} + 7\vec{k}$$

7.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & 1 & -1 \\ 1 & -4 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ -4 & 2 \end{vmatrix} \vec{\imath} - \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} \vec{\jmath} + \begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix} \vec{k} = -2\vec{\imath} - 7\vec{\jmath} - 13\vec{k}$$

$$(\vec{a} \times \vec{b}) \cdot \vec{a} = -2(3) - 7(1) - 13(-1) = 0 \checkmark \qquad (\vec{a} \times \vec{b}) \cdot \vec{b} = -2(1) - 7(-4) - 13(2) = 0 \checkmark$$

10. In the direction of

$$\begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & 5 & 2 \\ 1 & -3 & -2 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ -3 & -2 \end{vmatrix} \vec{\imath} - \begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} \vec{\jmath} + \begin{vmatrix} 3 & 5 \\ 1 & -3 \end{vmatrix} \vec{k} = -4\vec{\imath} + 8\vec{\jmath} - 14\vec{k}$$

12. Two vectors in (i.e., parallel to) the plane are $\vec{u} = (-2-3)\vec{\imath} + (1-4)\vec{\jmath} + 0 - 2\vec{k} = -5\vec{\imath} - 3\vec{\jmath} - 2\vec{k}$ and $\vec{v} = (0-3)\vec{\imath} + (2-4)\vec{\jmath} + 1 - 2\vec{k} = -3\vec{\imath} - 2\vec{\jmath} - \vec{k}$, so a vector perpendicular to it is

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ -5 & -3 & -2 \\ -3 & -2 & -1 \end{vmatrix} = -\vec{\imath} + \vec{\jmath} + \vec{k}$$

So the plane is $-(x-3) + (y-4) + (z-2) = 0$, or $-x + y + z = 3$.

14. Such a vector is perpendicular to the normals of both of the two planes, so one that works is

$$= \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & -3 & 5 \\ 4 & 1 & -3 \end{vmatrix} = 4\vec{\imath} + 26\vec{\jmath} + 14\vec{k}$$

15. $4x + 26y + 14z = 0$