

Problems 13.1, Page 102

2.

$$\begin{aligned} f_x(1, 3) &\approx \frac{f(1.1, 3) - f(1, 3)}{0.1} = \frac{e^{-1.1} \sin 3 - e^{-1} \sin 3}{0.1} \approx -.0494 \\ f_y(1, 3) &\approx \frac{f(1, 3.1) - f(1, 3)}{0.1} = \frac{e^{-1} \sin 3.1 - e^{-1} \sin 3}{0.1} \approx -.3662 \\ f_x(1, 3) &\approx \frac{f(1.1, 3) - f(1, 3)}{0.01} = \frac{e^{-1.01} \sin 3 - e^{-1} \sin 3}{0.01} \approx -.0517 \\ f_y(1, 3) &\approx \frac{f(1, 3.01) - f(1, 3)}{0.01} = \frac{e^{-1} \sin 3.01 - e^{-1} \sin 3}{0.01} \approx -.3645 \end{aligned}$$

(The solution book gives the following values: -0.0493 , -0.3660 , -0.0501 and -0.3629 respectively. I suspect roundoff error.)

6. (a) Concentration (maybe in milligrams of drug per milliliter of blood) per unit distance along the blood vessel. It describes how the concentration is changing as one looks at various points along the vessel, all at a given instant in time. Because that concentration is probably decreasing as the distance increases, I would guess its sign is negative.
 (b) Concentration per unit time since the injection. It describes how the concentration is changing at a certain point in the vessel at various times. The concentration rises at first and then falls, as the drug first flows from the injection site to the point at which the derivative is taken and then is eliminated from the body; so the derivative is probably positive at first and then negative.
8. (a) Positive: As x increases from A , z goes up.
 (b) Negative: As y increases from A , z goes down.
 (c) $f_x(P)$ starts positive, decreases to 0 (probably near where $x = 0$), and thereafter is more and more negative. $f_y(P)$ is always negative. It may be more negative near $x = 0$ — the surface looks a bit steeper in the y -direction there— but that is uncertain.
10. We will approximate $f_w(10, 25)$ by taking the average of the two difference quotients with w -values closest to $w = 10$, namely $w = 5$ and $w = 15$:

$$f_w(10, 25) \approx \frac{1}{2} \left(\frac{27 - 16}{5 - 10} + \frac{9 - 16}{15 - 10} \right) = -1.8,$$

so when the temperature is $25^\circ F$ and the wind speed is roughly 10 mph, each increase of 1 mph means that it feels 1.8° colder. (The solution book uses only one difference quotient, the one using $(15, 25)$, and gets an approximation of -1.6 .)

11. We will approximate $f_T(5, 20)$ by taking the average of the two difference quotients with T -values closest to $T = 20$, namely $T = 25$ and $T = 15$:

$$f_T(5, 20) \approx \frac{1}{2} \left(\frac{21 - 16}{25 - 20} + \frac{12 - 16}{15 - 20} \right) = 0.9,$$

so when the the wind speed is 5 mph and the temperature is roughly $20^\circ F$, each increase in (real) temperature of 1 degree means that it feels 0.9° warmer. (The solution book using only one difference quotient, the one using $(5, 25)$, and gets an approximation of 1.)