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2. $f_x = 6xy^2$, $f_y = 6x^2y + 2z$, $f_z = 2y$; so $f_x(-1,0,4) = 0$, $f_y(-1,0,4) = 8$, and $f_z(-1,0,4) = 0$.
   (a) $\vec{u} = (\vec{\imath} - \vec{k})/\sqrt{2}$, so $f_{\vec{u}}(-1,0,4) = 0$.
   (b) $\vec{u} = (-\vec{\imath} + 3\vec{\jmath} + 3\vec{k})/\sqrt{19}$, so $f_u(-1,0,4) = 8(3/\sqrt{19}) = 24/\sqrt{19}$.

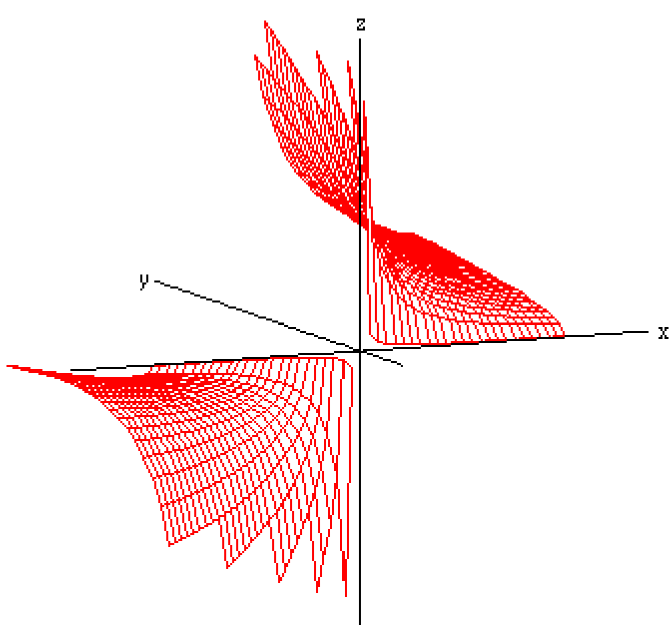
4. $f_x = -8(xy)^{-2}y = -8/(x^2y)$, and similarly $f_y = -8/(xy^2)$, so $f_x(1,2) = -4$ and $f_y(1,2) = -2$. So the desired tangent plane is $z = 4 - 4(x-1) - 2(y-2) = -4x - 2y + 12$.

6. (a) Let $F(x,y,z) = \cos x \sin y - z$. Then $F_x = -\sin x \sin y$, $F_y = \cos x \cos y$, and $F_z = -1$, so $F_x(0,\pi/2,1) = 0$, $F_y(0,\pi/2,1) = 0$, and $F_z(0,\pi/2,1) = -1$. So one normal vector is $\nabla F(0,\pi/2,1) = -\vec{k}$.
   (b) $0(x-0) + 0(y-\pi/2) - 1(z-1) = 0$, or $z = 1$ (horizontal).

7. (a) $f_x = (e^x - 1)\cos y$ and $f_y = -(e^x - x)\sin y$, so $f_x(2,3) = (e^2-1)\cos 3$ and $f_y(2,3) = -(e^2-2)\sin 3$. So one such desired vector is $-\nabla f(2,3) = -((e^2-1)\cos 3)\vec{\imath} + ((e^2-2)\sin 3)\vec{\jmath}$.
   (b) A normal to the surface at that point $((e^2-1)\cos 3)\vec{\imath} - ((e^2-2)\sin 3)\vec{\jmath} - \vec{k}$, so the given vector must be perpendicular to it, i.e., their dot product must be 0. Thus:

$$5((e^2-1)\cos 3) + 4(-(e^2-2)\sin 3) + a(-1) = 0 \implies a = 5(e^2-1)\cos 3 - 4(e^2-2)\sin 3 \approx -34.67\ .$$

11. It turns out that the point $(0,0,1)$ and the points in (a) are on the edge of the surface (see the graph below, where we are looking from behind the xz- and yz-planes — both pieces of the graph are "swooping" away from us, in a positive y-direction). As a result, partial derivatives, though they may be computable by formula, don't mean very much. So we will look only at the point $(1,1,1)$. Then, because $F_x = 2x$, $F_y = -z^{-2}$, and $F_z = 2yz^{-3}$, we get $\nabla F(1,1,1) = 2\vec{\imath} - \vec{\jmath} + 2\vec{k}$.
   (b) $2(x-1) - (y-1) + 2(z-1) = 0$, or $2x - y + 2z = 3$.
   (c) $\|\nabla F(1,1,1)\| = \sqrt{2^2 + (-1)^2 + 2^2} = 3$, so the desired unit vector (which the text calls $\vec{u}_2$, unfortunately) is $\frac{2}{3}\vec{\imath} - \frac{1}{3}\vec{\jmath} + \frac{2}{3}\vec{k}$.



12. Let $F(x,y,z) = 1 + x^2 + y^2 - z$; then $\nabla F = 2x\vec{\imath} + 2y\vec{\jmath} - 1\vec{k}$. Thus, to have the tangent plane to F be parallel to $z = 5$, ∇F must be normal to this plane, i.e., $2x = 2y = 0$, and this is true only at the point $(0,0,1)$. To have the tangent plane parallel to $z = 5 + 6x - 10y$, i.e., $6x - 10y - z = -5$, we must have the normal to this plane, $6\vec{\imath} - 10\vec{\jmath} - 1\vec{k}$, be a multiple of ∇F; and because the coefficients of $\vec{k}$ in both

are -1, they must be equal. So we need $2x = 6$ and $2y = -10$, i.e., $x = 3$ and $y = -5$. In this case, $z = 1 + 3^2 + (-5)^2 = 35$. So the point in question is $(3, -5, 35)$.

13. $z = 3 + 2(x - 4) - (y - 1)$, or $z = 2x - y - 4$.

14. (a) $2(x - 1) - 5(y - 3) = 0$, or $2x - 5y = -13$.
(b) $2(x - 1) - 5(y - 3) - (z - 7) = 0$, or $2x - 5y - z = -20$.

15. We note first that at least both surfaces contain the point in question (in both, when $x = 4$ and $y = 3$, we get $z = 5$); so we just need to verify that the tangent planes to each surface at that point are parallel to each other, i.e., that they have normal vectors that are parallel to each other, i.e., multiples of each other. Let $F(x, y, z) = 2x^2 + 2y^2 - 25 - z^2$ (the partial derivatives will be simpler than for $\sqrt{2x^2 + 2y^2 - 25} - z$, and it is clearly the same surface, except that it also allows negative z-values — and we are interested in a positive one) and $G(x, y, z) = \frac{1}{5}(x^2 + y^2) - z$. Then $\nabla F = 4x\vec{\imath} + 4y\vec{\jmath} - 2z\vec{k}$, so $\nabla F(4, 3, 5) = 16\vec{\imath} + 12\vec{\jmath} - 10\vec{k}$; and $\nabla G = \frac{2}{5}x\vec{\imath} + \frac{2}{5}y\vec{\jmath} - \vec{k}$, so $\nabla G(4, 3, 5) = \frac{8}{5}\vec{\imath} + \frac{6}{5}\vec{\jmath} - \vec{k}$. Therefore $\nabla F(4, 3, 5) = 10\nabla G(4, 3, 5)$, so the two surfaces have the same tangent plane through $(4, 3, 5)$.