

Problems 13.6, Page 140

2.

$$\frac{dz}{dt} = (\sin y + y \cos x)(2t) + (x \cos y + \sin x) \frac{1}{t} = 2t(\sin(\ln t) + (\ln t) \cos t^2) + \frac{t^2 \cos(\ln t) + \sin t^2}{t}$$

3.

$$\frac{dz}{dt} = \frac{1}{x^2 + y^2} (2x)(-t^{-2}) + \frac{1}{x^2 + y^2} (2y) \left(\frac{1}{2} t^{-1/2}\right) = \frac{-2t^{-3}}{t^{-2} + t} + \frac{1}{t^{-2} + t} = \frac{-2 + t^3}{t + t^4}$$

4.

$$\begin{aligned} \frac{dz}{dt} &= (\cos(x/y))(1/y)(2) + (\cos(x/y))(-xy^{-2})(-2t) \\ &= \left(\frac{2}{1-t^2} + \frac{4t^2}{(1-t^2)^2} \right) \cos\left(\frac{2t}{1-t^2}\right) = \left(\frac{2+2t^2}{(1-t^2)^2} \right) \cos\left(\frac{2t}{1-t^2}\right) \end{aligned}$$

6.

$$\frac{dz}{dt} = e^y(2) + [(x+y)e^y + e^y](-2t) = 2e^{1-t^2}(1-t[2t+1-t^2+1]) = 2e^{1-t^2}(t^3 - 2t^2 - 2t + 1)$$

7. I decided not to assign this one, but after typing the solution in all its gory detail, I couldn't bring myself to delete it:

$$\begin{aligned} \frac{\partial z}{\partial u} &= (e^{-y} - ye^{-x})(\sin v) + (-xe^{-y} + e^{-x})(-v \sin u) \\ &= (e^{-v \cos u} - (v \cos u)e^{-u \sin v})(\sin v) + (-(u \sin v)e^{-v \cos u} + e^{-u \sin v})(-v \sin u) \\ \frac{\partial z}{\partial v} &= (e^{-y} - ye^{-x})(u \cos v) + (-xe^{-y} + e^{-x})(\cos u) \\ &= (e^{-v \cos u} - (v \cos u)e^{-u \sin v})(u \cos v) + (-(u \sin v)e^{-v \cos u} + e^{-u \sin v})(\cos u) \end{aligned}$$

10.

$$\begin{aligned} \frac{\partial z}{\partial u} &= e^y \frac{1}{u} + [(x+y)e^y + e^y]0 = \frac{e^y}{u} \\ \frac{\partial z}{\partial v} &= e^y(0) + [(x+y)e^y + e^y]1 = e^y[\ln u + v + 1] \end{aligned}$$

11.

$$\begin{aligned} \frac{\partial z}{\partial u} &= e^y(2u) + xe^y(2u) = 2ue^{u^2-v^2}(1+u^2+v^2) \\ \frac{\partial z}{\partial v} &= e^y(2v) + xe^y(-2v) = 2ve^{u^2-v^2}(1-u^2-v^2) \end{aligned}$$

14.

$$\begin{aligned} \frac{\partial z}{\partial u} &= \frac{1}{1+(x/y)^2} (1/y)(2u) + \frac{1}{1+(x/y)^2} (-xy^{-2})(2u) \\ &= \frac{y}{y^2+x^2} (2u) + \frac{-x}{y^2+x^2} (2u) = \frac{y-x}{y^2+x^2} (2u) \\ &= 2u \frac{u^2-v^2-u^2-v^2}{u^4-2u^2v^2+v^4+u^4-2u^2v^2+v^4} = \frac{-4uv^2}{2u^4+2v^4} = \frac{-2uv^2}{u^4+v^4} \\ \frac{\partial z}{\partial v} &= \frac{y}{y^2+x^2} (2v) + \frac{-x}{y^2+x^2} (-2v) \\ &= \frac{y+x}{y^2+x^2} (2v) = 2v \frac{2u^2}{2u^4+2v^4} = \frac{2u^2v}{u^4+v^4} \end{aligned}$$