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3.

$$\begin{aligned} f_x &= e^y \\ f_{xx} &= 0 \\ f_{xy} &= e^y \\ f_y &= xe^y \\ f_{yx} &= e^y \\ f_{yy} &= xe^y \end{aligned}$$

So in particular $f_{xy} = f_{yx}$.

6.

$$\begin{aligned} f_x &= \frac{1}{2}(x^2+y^2)^{-1/2}(2x) = x(x^2+y^2)^{-1/2} \\ f_{xx} &= x(-\frac{1}{2}(x^2+y^2)^{-3/2}(2x)) + (x^2+y^2)^{-1/2} = -x^2(x^2+y^2)^{-3/2} + (x^2+y^2)^{-1/2} \\ &= (-x^2+(x^2+y^2))(x^2+y^2)^{-3/2} = y^2(x^2+y^2)^{-3/2} \\ f_{xy} &= x(-\frac{1}{2}(x^2+y^2)^{-3/2}(2y)) = -xy(x^2+y^2)^{-3/2} \\ f_y &= \frac{1}{2}(x^2+y^2)^{-1/2}(2y) = y(x^2+y^2)^{-1/2} \\ f_{yx} &= y(-\frac{1}{2}(x^2+y^2)^{-3/2}(2x)) = -xy(x^2+y^2)^{-3/2} \\ f_{yy} &= y(-\frac{1}{2}(x^2+y^2)^{-3/2}(2y)) + (x^2+y^2)^{-1/2} = -y^2(x^2+y^2)^{-3/2} + (x^2+y^2)^{-1/2} \\ &= (-y^2+(x^2+y^2))(x^2+y^2)^{-3/2} = x^2(x^2+y^2)^{-3/2} \end{aligned}$$

So in particular $f_{xy} = f_{yx}$.

7.

$$\begin{aligned} f_x &= (\cos(x/y))(y^{-1}) \\ f_{xx} &= (-\sin(x/y))(y^{-2}) \\ f_{xy} &= (\cos(x/y))(-y^{-2}) + (-\sin(x/y))(-xy^{-2})(y^{-1}) = -(\cos(x/y))(y^{-2}) + (\sin(x/y))(xy^{-3}) \\ f_y &= (\cos(x/y))(-xy^{-2}) \\ f_{yx} &= (\cos(x/y))(-y^{-2}) + (-\sin(x/y))(y^{-1})(-xy^{-2}) = -(\cos(x/y))(y^{-2}) + (\sin(x/y))(xy^{-3}) \\ f_{yy} &= (\cos(x/y))(2xy^{-3}) + (-\sin(x/y))(-xy^{-2})(-xy^{-2}) = 2(\cos(x/y))(xy^{-3}) - (\sin(x/y))(x^2y^{-4}) \end{aligned}$$

So in particular $f_{xy} = f_{yx}$.

9. $z_y = g(x)$, which has no y in it, so $z_{yy} = 0$.

10. (a) $z_{yx} = z_{xy} = 4y$, unless z is nasty.
(b) $z_{xyx} = \partial(4y)/\partial x = 0$.
(c) $z_{xyy} = \partial(4y)/\partial y = 4$.