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2. The quadratic Taylor polynomials for $\sin 2x$ and $\cos y$ are $2x$ and $1 - \frac{1}{2}y^2$, so the desired polynomial is $1 + 2x - \frac{1}{2}y^2$.

3. The quadratic Taylor polynomial for $\ln(1+t)$ is $t - \frac{1}{2}t^2$, so substituting $x^2 - y$ for t and dropping the terms of degree greater than 2, we get the desired polynomial: $-y + x^2 - \frac{1}{2}y^2$.

7.
$$\begin{array}{ll}
z = (x^2+y^2)^{1/2} & z(1,1) = \sqrt{2} \\
z_x = x(x^2+y^2)^{-1/2} & z_x(1,1) = 1/\sqrt{2} \\
z_y = y(x^2+y^2)^{-1/2} & z_y(1,1) = 1/\sqrt{2} \\
z_{xx} = -x^2(x^2+y^2)^{-3/2} + (x^2+y^2)^{-1/2} = y^2(x^2+y^2)^{-3/2} & z_{xx}(1,1) = 1/(2\sqrt{2}) \\
z_{xy} = -xy(x^2+y^2)^{-3/2} & z_{xy}(1,1) = -1/(2\sqrt{2}) \\
z_{yy} = x^2(x^2+y^2)^{-3/2} & z_{yy}(1,1) = 1/(2\sqrt{2})
\end{array}$$
so the desired linear polynomial is $\sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$ and the desired quadratic polynomial is $\sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1) + \frac{1}{4\sqrt{2}}(x-1)^2 - \frac{1}{2\sqrt{2}}(x-1)(y-1) + \frac{1}{4\sqrt{2}}(y-1)^2$. The "exact" value of $z(1.1, 1.1) = \sqrt{2.42}$, or at least as close as my calculator can come, is 1.55563491861. And

$$\sqrt{2} + \frac{1}{\sqrt{2}}(.1) + \frac{1}{\sqrt{2}}(.1) = \sqrt{2} + .1\sqrt{2} \approx 1.55563491861$$

$$\sqrt{2} + \frac{1}{\sqrt{2}}(.1) + \frac{1}{\sqrt{2}}(.1) + \frac{1}{4\sqrt{2}}(.1)^2 - \frac{1}{2\sqrt{2}}(.1)(.1) + \frac{1}{4\sqrt{2}}(.1)^2 = \sqrt{2} + .1\sqrt{2} \approx 1.55563491861$$

17. Note first that, because the error bound formulas in the text are phrased in terms of circular (rather than rectangular) regions around the point, in order to be sure that an error bound works on all of the rectangle — actually the square — $-0.1 < x < 0.1$, $-0.1 < y < 0.1$, we have to make M_L and M_Q bound the second and third partial derivatives on the circle with radius $0.1(\sqrt{2})$ about the origin, in which the x-values may run from $-0.1(\sqrt{2}) \approx -0.142$ to 0.142; and similarly for the y-values.
(a) Using the standard Taylor series ($e^x - x = (1 + x + \frac{1}{2}x^2 \ldots) - x$ and $\cos y = 1 - \frac{1}{2}y^2 + \ldots$) and dropping the terms of degree greater than 1, we get $L(x,y) = 1$ — there are no degree-1 terms.
    Now, $f_{xx} = e^x \cos y$,$f_{xy} = -(e^x - 1)\sin y$, and $f_{yy} = -(e^x - x)\cos y$. The largest absolute value that the the first factors (involving x: e^x, $e^x - 1$, $e^x - x$) ever takes on when $-0.142 < x < 0.142$ is $e^{0.142} < 1.16$; and the largest absolute value that the second factors (involving y: $\cos y$ or $\sin y$) take on when $-0.142 < y < 0.142$ is $\cos 0 = 1$. So the second partials are bounded on the circle of radius $0.1(\sqrt{2})$ about the origin by $M_L = (1.16)1 = 1.16$. By the formula given in the text, $|E_L(x,y)| \leq 2(1.16)(0.1^2 + 0.1^2) = 0.0464$ on $-0.1 < x < 0.1$, $-0.1 < y < 0.1$.
(b) Using the standard Taylor series and dropping the terms of degree greater than 2, we get $Q(x,y) = 1 + \frac{1}{2}x^2 - \frac{1}{2}y^2$.
    Now, $f_{xxx} = e^x \cos x$, $f_{xxy} = -e^x \sin y$, $f_{xyy} = -(e^x - 1)\cos y$, and $f_{yyy} = (e^x - x)\sin y$, so by reasoning similar to that in (a), the absolute values of third partials are also bounded on the circle of radius $0.1(\sqrt{2})$ about the origin by $M_Q = 1.16$. By the formula given in the text, $|E_Q(x,y)| \leq \frac{4}{3}(1.16)(0.1^2 + 0.1^2)^{3/2} < 0.0044$ on $-0.1 < x < 0.1$, $-0.1 < y < 0.1$.
(c)

$$\begin{aligned}
f(0.1, 0.1) &= (e^{0.1} - 0.1)\cos(0.1) \approx 1.0001492503 \\
L(0.1, 0.1) &= 1, \qquad \text{so} \qquad E_L(0.1, 0.1) \approx 0.0001492503 \\
Q(0.1, 0.1) &= 1 + \frac{1}{2}(0.1)^2 - \frac{1}{2}(0.1)^2 = 1, \qquad \text{so} \qquad E_Q(0.1, 0.1) \approx 0.0001492503, \text{ also.}
\end{aligned}$$