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2. (a) Local maximum (b) Saddle point (c) Local minimum. (d) None of these

7. $f_x = (x+y)y + (xy+1)1 = 2xy + y^2 + 1$ and $f_y = (x+y)x + (xy+1)1 = x^2 + 2xy + 1$, so where both are equal to 0, we have $x^2 = -2xy - 1 = y^2$, so $x = \pm y$. Taking the plus sign gives $3x^2 + 1 = 0$, so we get no such points; and from $y = -x$ we get $-x^2 + 1 = 0$, so $x = \pm 1$. So the critical points are $(1,-1)$ and $(-1,1)$. Now $f_{xx} = 2y$, $f_{xy} = 2x + 2y$, and $f_{yy} = 2x$. So at $(1,-1)$ we have $D = (-2)(2) - 0 < 0$, and at $(-1,1)$ we have $D = (2)(-2) - 0 < 0$, so both are saddle points.

8. $f_x = 8y - (x+y)^3$ and $f_y = 8x - (x+y)^3$, so where both are 0 we have $y = \frac{1}{8}(x+y)^3 = x$, and from $8x - 8x^3 = 0$ we see that the critical points are $(0,0)$, $(1,1)$ and $(-1,-1)$. Now $f_{xx} = -3(x+y)^2 = f_{yy}$, and $f_{xy} = 8 - 3(x+y)^2$, so in general $D = -(64 - 48(x+y)^2)$. Hence at $(0,0)$, $(1,1)$ and $(-1,-1)$ respectively, $D = -64$, 128 and 128. So $(0,0)$ is a saddle point, and because $f_{xx} < 0$ at $(1,1)$ and $(-1,-1)$, these are local maxima.

9. $E_x = \sin x$, and $E_y = 2y$, so both are 0 at $(\pi k, 0)$ where k is any integer. Now $E_{xx} = \cos x$, $E_{xy} = 0$, and $E_{yy} = 2$, so $D = 2\cos x$, which is negative at $x = \pi k$ when k is odd — so $(\pi k, 0)$ is a saddle point when k is odd — and positive when k is even — and because E_{xx} is also positive there, $(\pi k, 0)$ is a local minimum when k is even.