Problems 15.2, Page 233

These solutions include several problems that were not assigned: The original assignment was too long, but I had worked so hard typing the solutions that I couldn't bring myself to delete them.

6. The line through $(-1, 1)$ and $(3, -2)$ is $y - 1 = \frac{-2-1}{3-(-1)}(x - (-1))$ or $y = -\frac{3}{4}x + \frac{1}{4} = (1 - 3x)/4$, so the desired integral can be written
$$\int_{-1}^{3} \int_{-2}^{(1-3x)/4} f\, dy\, dx\ .$$

7. The line through $(1, 0)$ and $(4, 1)$ is $y - 0 = \frac{1-0}{4-1}(x - 1)$ or $y = \frac{1}{3}x - \frac{1}{3} = (x - 1)/3$, so the desired integral can be written
$$\int_{1}^{4} \int_{(x-1)/3}^{2} f\, dy\, dx\ .$$

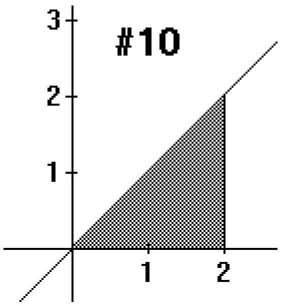
8. The line through $(0, 1)$ and $(1, 3)$ is $y - 1 = \frac{3-1}{1-0}(x - 0)$ or $x = (y - 1)/2$, and the one through $(1, 3)$ and $(2, 1)$ is $y - 3 = \frac{1-3}{2-1}(x - 1)$ or $x = 1 - \frac{1}{2}(y - 3) = (5 - y)/2$, so the desired integral can be written
$$\int_{1}^{3} \int_{(y-1)/2}^{(5-y)/2} f\, dy\, dx\ .$$

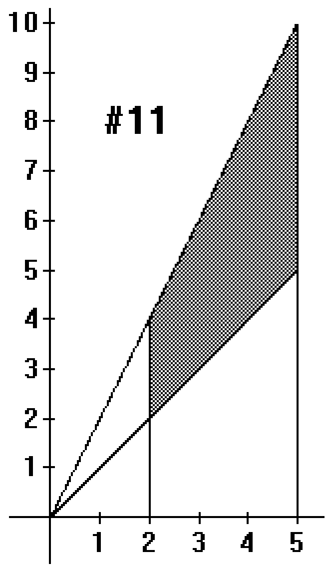
10. The diagrams of the regions of integration for the next several problems are drawn below. In the middle of the next computation, we find that the substitution $u = x^2$ is useful (so that $du = 2x\, dx$ and as x varies from 0 to 2, u goes from 0 to 4):
$$\begin{aligned}\int_0^2 \int_0^x e^{x^2}\, dy\, dx &= \int_0^2 e^{x^2} \int_0^x dy\, dx = \int_0^2 e^{x^2}\, [y]_0^x\, dx = \int_0^2 e^{x^2} x\, dx \\ &= \frac{1}{2}\int_0^4 e^u\, du = \frac{1}{2}e^u\bigg|_0^4 = \frac{e^4 - 1}{2} \approx 26.8\end{aligned}$$

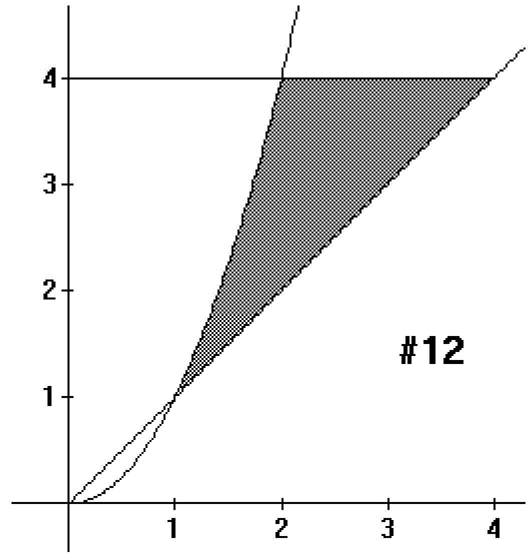
11. In the middle of the next computation, we find that integration by parts, with $u = x$, $dv = \sin x\, dx$, is useful:
$$\begin{aligned}\int_1^5 \int_x^{2x} \sin(x)\, dy\, dx &= \int_1^5 \sin(x) \int_x^{2x} dy\, dx = \int_1^5 \sin(x)\, [y]_x^{2x}\, dx = \int_1^5 x\sin(x)\, dx \\ &= [-x\cos(x)]_1^5 + \int_1^5 \cos(x)\, dx = -5\cos 5 + \cos 1 + [\sin(x)]_1^5 \\ &= -5\cos 5 + \cos 1 + \sin 5 - \sin 1 \approx -2.68\end{aligned}$$

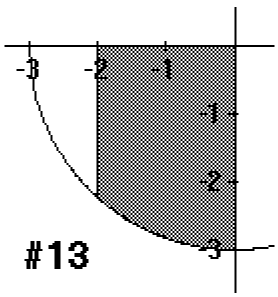
12.
$$\begin{aligned}\int_1^4 \int_{\sqrt{y}}^{y} x^2y^3, dx\, dy &= \int_1^4 y^3[\frac{1}{3}x^3]_{\sqrt{y}}^{y}\, dx\, dy = \frac{1}{3}\int_1^4 (y^6 - y^{9/2})\, dy = \frac{1}{3}\left[\frac{1}{7}y^7 - \frac{2}{11}y^{11/2}\right]_1^4 \\ &= \frac{1}{3}(\frac{1}{7}(4^7 - 1) - \frac{2}{11}(2^{11} - 1)) = \frac{151,555}{231} \approx 656.1\end{aligned}$$

13.
$$\begin{aligned}\int_{-2}^0 \int_{-\sqrt{9-x^2}}^{0} 2xy\, dy\, dx &= \int_{-2}^0 x[y^2]_{-\sqrt{9-x^2}}^{0}\, dx = \int_{-2}^0 x(0 - (9 - x^2))\, dx = \int_{-2}^0 (x^3 - 9x)\, dx \\ &= \left[\frac{1}{4}x^4 - \frac{9}{2}x^2\right]_{-2}^0 = 0 - (\frac{(-2)^4}{4} - \frac{9}{2}(-2)^2) = -(4 - 18) = 14\end{aligned}$$









14. (a) See below.
    (b) Rewriting $x = -(y-4)/2$ gives $y = 4 - 2x$, so the desired integral is

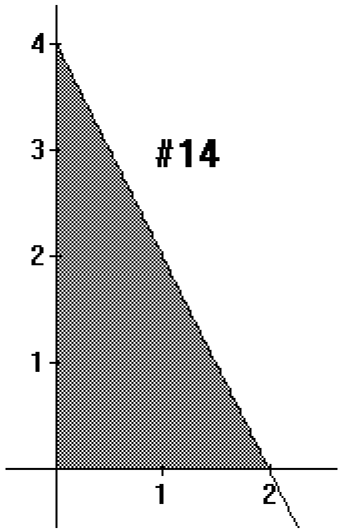
$$\int_0^2 \int_0^{4-2x} g(x)\, dy\, dx \ .$$

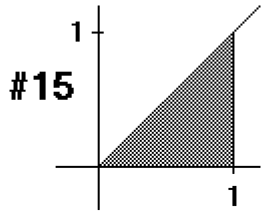
15.

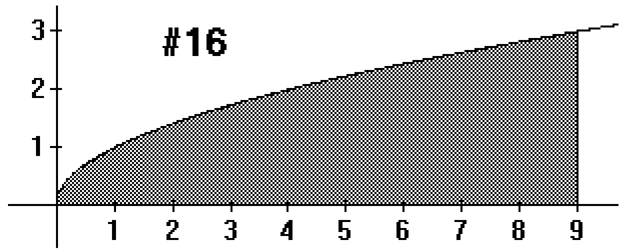
$$\int_0^1 \int_y^1 e^{x^2}\, dx\, dy = \int_0^1 \int_0^x e^{x^2}\, dy\, dx = \int_0^1 x e^{x^2}\, dx = \frac{1}{2} e^{x^2}|_0^1 = \frac{e-1}{2} \ .$$

16.

$$\begin{aligned}
\int_0^3 \int_{y^2}^9 y \sin(x^2)\, dx\, dy &= \int_0^9 \int_0^{\sqrt{x}} y \sin(x^2)\, dx\, dy = \int_0^9 \sin(x^2) \int_0^{\sqrt{x}} y\, dy\, dx \\
&= \int_0^9 \sin(x^2)[\frac{1}{2}y^2]_0^{\sqrt{x}}\, dx = \frac{1}{2}\int_0^9 x \sin(x^2)\, dx \\
&= -\frac{1}{4}\cos(x^2)|_0^9 = \frac{1}{4}(1 - \cos 81)
\end{aligned}$$







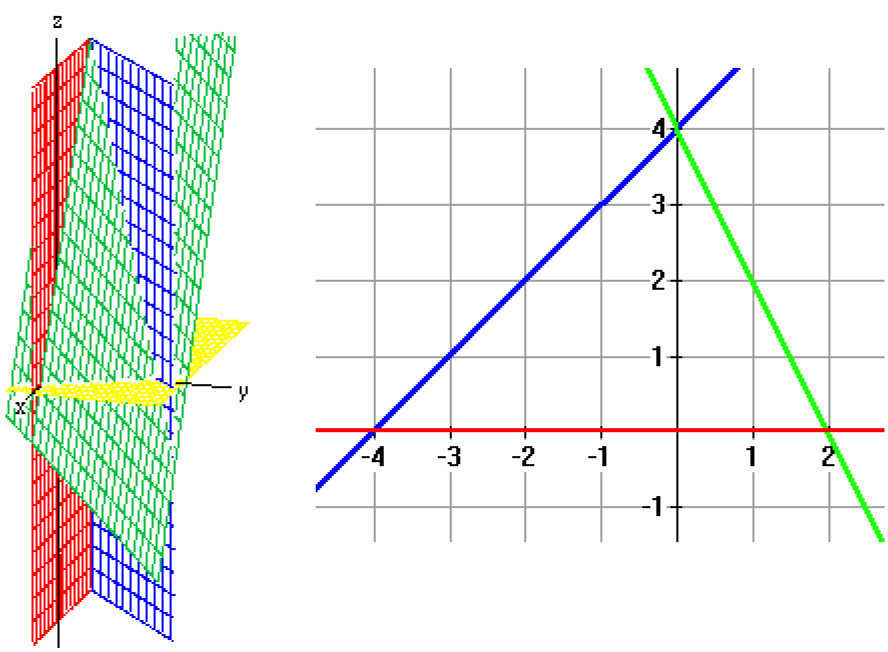
19. The surface cuts the xy-plane (i.e., $z = 0$) in the circle $0 = 25 - x^2 - y^2$, or $x^2 + y^2 = 25$, so we want to integrate over the interior of that circle:

$$\int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} (25 - x^2 - y^2)\, dy\, dx \ .$$

20. The surface cuts the plane $z = 16$ in the circle $25 - x^2 - y^2 = 16$, or $x^2 + y^2 = 9$, so the region of integration is the interior of that circle:

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (25 - x^2 - y^2 - 16)\, dy\, dx = \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (9 - x^2 - y^2)\, dy\, dx \ .$$

21.



The two vertical planes meet the xy-plane (i.e., $z = 0$) in the x-axis and the line $y = x + 4$ respectively, and the slanted plane meets the xy-plane in the line $2x + y = 4$. So the region of integration is the interior of the triangle formed by those three lines. The other two lines meet the x-axis (i.e., $y - 0$) at $(-4, 0)$ and $(2, 0)$; and they meet each other at $(0, 4)$. For a fixed y-value in this triangle, by solving the other two lines for x, we see that the x-values run from $y - 4$ to $(4 - y)/2$. So the desired integral is

$$\int_0^4 \int_{y-4}^{(4-y)/2} (4 - 2x - y)\, dx\, dy\ .$$