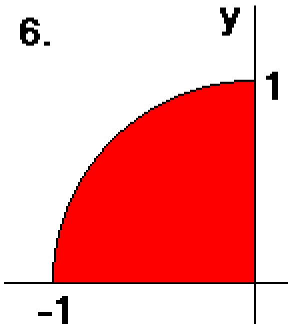
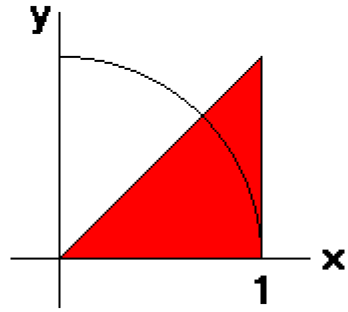
Problems 15.5, Page 246

2. $\displaystyle\int_{\pi/2}^{3\pi/2}\int_1^2 fr\,dr\,d\theta$ 3. $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{2}} fr\,dr\,d\theta$ 4. $\displaystyle\int_0^{\pi/2}\int_0^{0.5} fr\,dr\,d\theta$

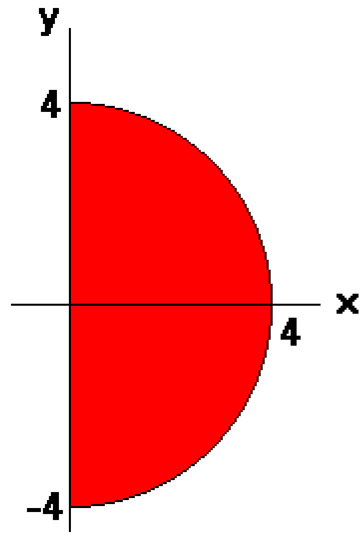
6.



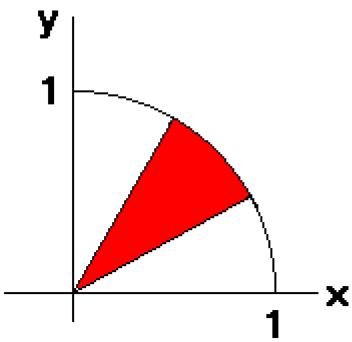
9.



11.



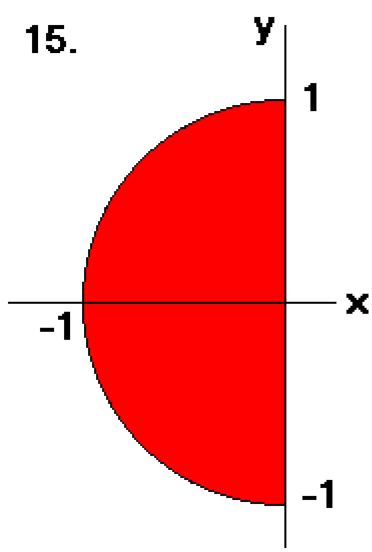
7.



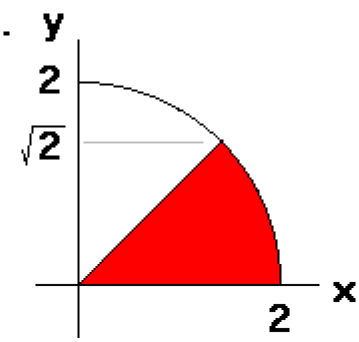
12. With the substitution $u = r^2$ (so that $du = 2r\,dr$ and, as r varies from 0 to 2, u varies from 0 to 4), we get

$$\int_0^{2\pi}\int_0^2 (\sin r^2) r\,dr\,d\theta = \int_0^{2\pi}\left[-\frac{1}{2}\cos u\right]_0^4 d\theta = \int_0^{2\pi}\frac{1}{2}(1-\cos 4)\,d\theta = \pi(1-\cos 4)\ .$$

15.



16.



The graphs for #15 and #16:

15.

$$\int_{\pi/2}^{3\pi/2}\int_0^1 (r\cos\theta)r\,dr\,d\theta = \left(\int_0^1 r^2\,dr\right)\left(\int_{\pi/2}^{3\pi/2}\cos\theta\,d\theta\right) = \left(\left[\frac{1}{3}r^3\right]_0^1\right)\left([\sin\theta]_{\pi/2}^{3\pi/2}\right) = -\frac{2}{3}$$

16.

$$\begin{aligned}\int_0^{\pi/4}\int_0^2 (r\cos\theta)(r\sin\theta)r\,dr\,d\theta &= \left(\int_0^2 r^3\,dr\right)\left(\int_0^{\pi/4}\cos\theta\,\sin\theta\,d\theta\right)\\ &= \left(\left[\frac{1}{4}r^4\right]_0^2\right)\left(\left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/4}\right) = 4\left(\frac{1}{4}\right) = 1\end{aligned}$$

19. The two surfaces meet where $\sqrt{8-x^2-y^2} = \sqrt{x^2+y^2}$, or $x^2+y^2=4$, the circle with radius 2 centered at $x=0$, $y=0$ (and $z=2$), so the solid exists over the disc of radius 2 centered at the origin in the xy-plane. So:

$$\begin{aligned}\text{Volume} &= \int_0^{2\pi}\int_0^2 (\sqrt{8-r^2}-r)r\,dr\,d\theta = \left(\int_0^2 (\sqrt{8-r^2}-r)r\,dr\right)\left(\int_0^{2\pi} d\theta\right)\\ &= \left(\left[-\frac{1}{3}(8-r^2)^{3/2}-\frac{1}{3}r^3\right]_0^2\right)\left([\theta]_0^{2\pi}\right) = \left(\left(-\frac{8}{3}-\frac{8}{3}\right)+\frac{\sqrt{512}}{3}\right)2\pi = \frac{2}{3}\pi(\sqrt{512}-16)\ .\end{aligned}$$

21. (a)

$$\text{Population} = \int_{\pi/2}^{3\pi/2}\int_1^4 \delta(r,\theta)r\,dr\,d\theta$$

(b) We want δ to decrease as r increases, so we want the factor of δ with r in it to be $4-r$; and as θ moves from $\pi/2$ to π and then to $3\pi/2$, we want δ first to decrease and then to increase, so we want the factor of δ with θ in it to be $2+\cos\theta$. So the best answer of these three is (i).
(c)

$$\begin{aligned}\text{Population} &= \int_{\pi/2}^{3\pi/2}\int_1^4 (4-r)(2+\cos\theta)r\,dr\,d\theta = \left(\int_1^4 (4-r)r\,dr\right)\left(\int_{\pi/2}^{3\pi/2}(2+\cos\theta)\,d\theta\right)\\ &= \left(\left[2r^2-\frac{1}{3}r^3\right]_1^4\right)\left([2\theta+\sin\theta]_{\pi/2}^{3\pi/2}\right) = (30-21)\,(2\pi+(-1-1)) = 18(\pi-1)\approx 38.5\end{aligned}$$