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2. In 4 time units the point traces around a rectangle from (0,0) to (2,0) to (2,1) to (0,1) and back to (0,0), taking one time unit per side (so it travels faster horizontally than vertically).

3. In 4 time units the point traces around a re-entrant quadrilateral (two opposite sides cross at the origin) from $(-1,1)$ to $(1,1)$ to $(-1,-1)$ to $(1,-1)$ and back to $(-1,1)$, taking one unit per side (so it travels faster diagonally than horizontally).

11. In (a) and (c) the point traces out the parabola $y = x^2$, left to right; in (c) it moves more slowly near (0,0) and faster for large negative and large positive values of t than in (a). In (b) it traces out only the right half of this parabola, sliding down the right half, stopping at (0,0) when $t = 0$, and moving back up the right half.

12. One answer is $x = 3\cos t$, $y = -3\sin t$.

13. One answer is $x = -2$, $y = t$.

14. One answer is $x = 2 + 5\cos t$, $y = 1 + 5\sin t$.

16. One answer is $x = 2 + (1-2)t = 2 - t$, $y = -1 + (3-(-1))t = 4t - 1$.

17. One answer is $x = 5\cos t$, $y = 7\sin t$.

24. One answer is $x = 3\cos t$, $y = 3\sin t$, $z = 2$.

25. One answer is $x = 2 + (5-2)t = 2 + 3t$, $y = 3 + (2-3)t = 3 - t$, $z = -1 + (0-(-1))t = t - 1$.

26. One answer is $x = 1 + 3t$, $y = 2 - 3t$, $z = 3 + t$.

30. The t in the two expressions we gave need not be the same, so we are really asking whether the system of equations
$$2 + 3s = 1 + 3t \qquad 3 - s = 2 - 3t \qquad 3 - s = 3 + t$$
has a common solution. Adding the first two equations gives $5 + 2s = 3$, so if there is a solution, it must have $s = -1$, and substituting this into the second gives $t = -2/3$. But substituting these two values into the third equation gives a false statement; so the two lines do not meet.

31. The line through the two given points is $x = -3 + 7t$, $y = -4 + 9t$, $z = 2 - 2t$, so the question is whether there is any point on this line that also lies on the sphere; i.e., whether the equation $(-3+7t)^2 + (-4+9t)^2 + (2-2t)^2 = 1$ has a solution. Rewriting this equation gives $134t^2 - 122t + 28 = 0$, and the discriminant is $(-122)^2 - 4(134)(28) < 0$. Thus, the equation has no real roots, the line does not pass through the opaque sphere, and each point is visible from the other.

32. As required, for all t, we have $(3+t) + (2t) + 3(1-t) = 6$ and $(3+t) - (2t) - (1-t) = 2$. Thus, this line lies in both planes (and hence is their intersection, because two different planes cannot intersect in more than one line).

33. (a) Both are lines. The first goes through $(-1,4,-1)$ and $(0,3,1)$; the second goes through $(-7,-6,-1)$ and $(-5,-4,0)$.
(b) The question is whether there is a time t at which the particles have the same coordinates. They have the same x-coordinate only when $-1 + t = -7 + 2t$, or $t = 6$, and at that time their y-coordinates are -2 and 6 respectively; so the particles do not collide.
(c) The question is whether there is a common solution to the system
$$-1 + t = -7 + 2s \qquad 4 - t = -6 + 2s \qquad -1 + 2t = -1 + s$$
Subtracting the first two equations gives $-5 - 2t = -1$ or $t = 2$, and substituting this into the first equation gives $s = 4$. These values also work in the third equation, so the two lines do cross, at the point $(-1+2, 4-2, -1+2(2)) = (1,2,3)$.