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3. (a) The normal vector to each plane is perpendicular to every line in that plane and so in particular to the line of intersection of the two planes. But the cross product of the two normal vectors is also perpendicular to both of them, so it must be parallel to the line of intersection.
(b) $(\vec{\imath} + 2\vec{\jmath} - 3\vec{k}) \times (3\vec{\imath} - \vec{\jmath} + \vec{k}) = -\vec{\imath} - 10\vec{\jmath} - 7\vec{k}$, by the usual determinant formula for cross product.
(c) We need one point on the line of intersection: Solving the first plane's equation for x gives $x = 7 - 2y + 3z$, and putting that into the second equation gives $21 - 6y + 9z - y + z = 0$ or $7y - 10z = 21$; and one such point is given by $z = 0$, $y = 3$, and then $x = 1$. So the desired equations might be

$$x = 1 - t\ , \qquad y = 3 - 10t\ , \qquad z = -7t\ .$$

9. $\vec{v}(t) = (2t-2)\vec{\imath} + (3t^2-3)\vec{\jmath} + (12t^3 - 12t^2)\vec{k}$, so $\|\vec{v}(t)\| = \sqrt{(2t-2)^2 + (3t^2-3)^2 + (12t^3-12t^2)^2} = |t-1|\sqrt{4 + 9(t+1)^2 + 144t^4}$, and the only time at which all the coordinates of $\vec{v}(t)$ are 0 is $t = 1$.

12. $\vec{v}(t) = \vec{\imath} + (3t^2 - 1)\vec{\jmath}$, $\vec{a}(t) = 6t\vec{\jmath}$

13. $\vec{v}(t) = 3\vec{\imath} + \vec{\jmath} - \vec{k}$, $\vec{a}(t) = \vec{0}$

16.
$$\int_0^1 \sqrt{(-e^t\,\sin(e^t))^2 + (e^t\,\cos(e^t))^2}\,dt = \int_0^1 e^t\,dt = e^t\Big|_0^1 = e - 1$$

18. $x = 5 + 2(t-4)$, $y = 4 - 3(t-4)$, $z = -2 + (t-4)$

23. (a) No: the coefficient of $\vec{k}$ in $\vec{v}$ is 2, which is positive.
(b) $2t = 10$ when $t = 5$. (We are not given time units.)
(c) $\vec{v}(t) = (-\sin t)\vec{\imath} + (\cos t)\vec{\jmath} + 2\vec{k}$, so $\vec{v}(5) = (-\sin 5)\vec{\imath} + (\cos 5)\vec{\jmath} + 2\vec{k}$.
(d) It leaves the helix at position $(\cos 5, \sin 5, 10)$. Letting $t = 0$ now denote the time at which it leaves the helix, $x = \cos 5 - (\sin 5)t$, $y = \sin 5 + (\cos 5)t$, $z = 10 + 2t$. (The answer book doesn't change $t = 5$ when the particle leaves the helix back to $t = 0$ again, and so the t's in these equations are replaced there by $(t-5)$'s.)

24. $\vec{v}(t) = (-6e^{3t})\vec{\imath} - (5\sin t)\vec{\jmath} - (6\cos(2t))\vec{k}$, so $\vec{r}(0) = -2\vec{\imath} + 5\vec{\jmath}$ and $\vec{v}(0) = -6\vec{\imath} - 6\vec{k}$. Thus, t sec after Skywalker turns off his thrusters, his position vector is given by $\vec{r}(t) = (-2-6t)\vec{\imath} + 5\vec{\jmath} - (6t)\vec{k}$. To get $-6t = 3.5$ means t is negative, so even though the Xardon station is on the tangent line ($t = -3.5/6$ gives $\vec{r}(-3.5/6) = 1.5\vec{\imath} + 5\vec{\jmath} + 3.5\vec{k}$, the position vector of the Xardon station), he won't get there, because he has waited too long and is already past it.

28. (a) Her position vector t sec after dropping the ball, relative to the center of the merry-go-round, with P at (10,0) on the x-axis, can be expressed as $\vec{r}(t) = (10\cos\frac{1}{20}(2\pi t))\vec{\imath} + (10\sin\frac{1}{20}(2\pi t))\vec{\jmath} = (10\cos(\frac{\pi}{10}t))\vec{\imath} + (10\sin(\frac{\pi}{10}t))\vec{\jmath}$, so her velocity is $\vec{v}(t) = -(\pi\sin(\frac{\pi}{10}t))\vec{\imath} + (\pi\cos(\frac{\pi}{10}t))\vec{\jmath}$ and her speed is $\|\vec{v}(t)\| = \sqrt{\pi^2\sin^2(\frac{\pi}{10}t) + \pi^2\cos^2(\frac{\pi}{10}t)} = \pi$ m/s.
(b) The distance the ball will fall in t sec is given by $\frac{1}{2}(9.8)t^2$, so by solving $\frac{1}{2}(9.8)t^2 = 3$, we see that the ball will take $t = \sqrt{6/9.8}$ sec to fall 3 m. The ball's horizontal speed is π m/s, so until it hits the ground, it will travel horizontally $\pi\sqrt{6/9.8} \approx 2.45$ m.
(c) The ball hits the ground at $(10, \pi\sqrt{6/9.8})$, and in the $\sqrt{6/9.8}$ sec since she dropped it, Emily has moved to $\vec{r}(\sqrt{6/9.8}) = (10\cos(\frac{\pi}{10}\sqrt{6/9.8}))\vec{\imath} + (10\sin(\frac{\pi}{10}\sqrt{6/9.8}))\vec{\jmath}$, so the distance between them is

$$\sqrt{\left(10 - 10\cos(\frac{\pi}{10}\sqrt{6/9.8})\right)^2 + \left(\pi\sqrt{6/9.8} - 10\sin(\frac{\pi}{10}\sqrt{6/9.8})\right)^2} \approx 0.302 \text{ m}$$

(This is the distance between the point at ground level under Emily's feet and the point where the ball hits the ground. The answer book finds the distance between the ball's impact point and Emily's dropping hand, 3 m high, and gets about 3.01 m.)