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4. $x' = y$, $y' = x$: $(a(e^t + e^{-t}))' = a(e^t - e^{-t})$ and $(a(e^t - e^{-t}))' = a(e^t + e^{-t})$ (where we are assuming that a is a constant); so this pair of functions of t satisfies the differential equations in x and y.

5. $x' = y$, $y' = -x$: $(a \sin t)' = a \cos t$ and $(a \cos t)' = -a \sin t$, so this pair of functions is a solution to the system of differential equations.

7. $x' = x$, $y' = -y$: $(ae^t)' = ae^t$ and $(be^{-t})' = -be^{-t}$, so this pair of functions is a solution to the systems of differential equations.

9. (a) From (1,0) the flow line goes straight up, but as soon as it leaves that point, there is a movement to the right: (III). The arrows are tricky: they point up on the through (1,0), right on the one through (0,1), down on the one through $(-1, 0)$, left on the one through $(0, -1)$, away from the origin on the two halves of the line $y = x$, and toward the origin on the two halves of the line $y = -x$.
   (b) From (1,0) the flow line goes straight up, but as soon as it leaves that point, there is a movement to the left: (I), with arrows counterclockwise.
   (c) From any point the movement is directly away from the origin: (II)
   (d) The movement is roughly circular counterclockwise, like (b), but whenever we are not exactly on the x-axis (e.g., at (0,5)), there is a small additional movement away from the x-axis: (V) (spiraling away from the origin).
   (e) The movement is roughly circular counterclockwise, like (b), but whenever we are not exactly on the x-axis (e.g., at (0,5)), there is a small additional movement toward the x-axis: (VI) (spiraling toward the origin).
   (f) By elimination: (IV). Or better, all the vectors in the field have slope 1, so all the flow lines have slope 1. When $x > y$, they go right and up; when $x < y$, they go down and left; so the line $x = y$ divides those that go in one direction from those that go in the other (and along that line the vector field is $\vec{0}$).