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3. We can parameterize C by $x = 1 + t$, $y = 2 + t$, $0 \le t \le 2$. Then $dx = 1dt$, $dy = 1dt$, and we have

$$\begin{aligned}\int_C \vec{F} \cdot d\vec{r} &= \int_0^2 \left((2+t)^2(1) + (1+t)^2(1)\right) dt = \int_0^2 (2t^2 + 6t + 5)dt \\ &= \left[\frac{2}{3}t^3 + 3t^2 + 5t\right]_0^2 = \frac{16}{3} + 12 + 10 = \frac{82}{3} \ .\end{aligned}$$

5. A parameterization for that part of the ellipse is $x = 2\sin t$, $y = \cos t$, $0 \le t \le \frac{\pi}{2}$. Then $dx = 2\cos t\, dt$ and $dy = -\sin t\, dt$, so

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi/2} (e^{2\sin t} 2\cos t - e^{\cos t}\sin t)dt = \left[e^{2\sin t} + e^{\cos t}\right]_0^{\pi/2} = e^2 + 1 - 1 - e = e(e-1) \ .$$

6. We need to find three parametrizations, one for each side of the triangle, going in the right direction.

For the side C_1 from $(-1, 0)$ to (0,1), we can use $x = t - 1$, $y = t$, $0 \le t \le 1$. Then $dx = dy = 1dt$ and

$$\begin{aligned}\int_{C_1} \vec{F} \cdot d\vec{r} &= \int_0^1 ((t-1)t(1) + (t-1-t)(1))\, dt = \int_0^1 (t^2 - t - 1)dt \\ &= \left[\frac{1}{3}t^3 - \frac{1}{2}t^2 - t\right]_0^1 = \frac{1}{3} - \frac{1}{2} - 1 = -\frac{7}{6} \ .\end{aligned}$$

For the side C_2 from (0,1) to (1,0), we can use $x = t$, $y = 1 - t$, $0 \le t \le 1$. Then $dx = 1dt$, $dy = -1dt$, and

$$\begin{aligned}\int_{C_2} \vec{F} \cdot d\vec{r} &= \int_0^1 (t(1-t)(1) + (t - (1-t))(-1))\, dt = \int_0^1 (-t^2 - t + 1)dt \\ &= \left[-\frac{1}{3}t^3 - \frac{1}{2}t^2 + t\right]_0^1 = -\frac{1}{3} - \frac{1}{2} + 1 = \frac{1}{6} \ .\end{aligned}$$

For the side C_3 from (1,0) to $(-1, 0)$, we can use $x = -t$, $y = 0$, $0 \le t \le 2$. Then $dx = -1dt$, $dy = 0$, and

$$\int_{C_3} \vec{F} \cdot d\vec{r} = \int_0^2 (-t(0)(-1) + (-t-0)(0))\, dt = 0 \ .$$

(We should have seen that coming: On C_3, the vectors from $\vec{F}$ have zero $\vec{\imath}$-components, so they are all perpendicular to C_3, so the line integral will be 0.)

Therefore,

$$\int_{C_3} \vec{F} \cdot d\vec{r} = -\frac{7}{6} + \frac{1}{6} + 0 = -1 \ .$$

7. $dx = dt$, $dy = 2t\, dt$, and $dz = 3t^2\, dt$, so

$$\begin{aligned}\int_{C_3} \vec{F} \cdot d\vec{r} &= \int_1^2 \left(t(1) + 2(t^3)(t^2)(2t) + t(3t^2)\right) dt = \int_1^2 (4t^6 + 3t^3 + t)dt \\ &= \left[\frac{4}{7}t^7 + \frac{3}{4}t^4 + \frac{1}{2}t^2\right]_1^2 = \frac{4}{7}(127) + \frac{3}{4}(15) + \frac{1}{2}(3) = \frac{4778}{56} \approx 85.3 \ .\end{aligned}$$