Problems 18.3, Page 364

2. Natural parameterizations are given by, for A: $x = y = t$, $0 \le t \le 1$; for B: $x = t$, $y = t^2$, $0 \le t \le 1$; and C: $x = t$, $y = 0$ for $0 \le t \le 1$ and $x = 1$, $y = t - 1$ for $1 \le t \le 2$:

$$\begin{aligned}
\int_A \vec{F} \cdot d\vec{r} &= \int_0^1 (t\vec{\imath} + t\vec{\jmath}) \cdot (dt\vec{\imath} + dt\vec{\jmath}) = 2\int_0^1 t\,dt = \left[t^2\right]_0^1 = 1 \\
\int_B \vec{F} \cdot d\vec{r} &= \int_0^1 (t\vec{\imath} + t^2\vec{\jmath}) \cdot (dt\vec{\imath} + 2t\,dt\vec{\jmath}) = \int_0^1 (t + 2t^3)dt = \left[\frac{1}{2}t^2 + \frac{1}{2}x^4\right]_0^1 = 1 \\
\int_C \vec{F} \cdot d\vec{r} &= \int_0^1 t\,dt + \int_1^2 (t-1)dt = \left[\frac{1}{2}t^2\right]_0^1 + \left[\frac{1}{2}(t-1)^2\right]_1^2 = \frac{1}{2} + \frac{1}{2} = 1
\end{aligned}$$

3. $\partial F_2/\partial x = 0$, $\partial F_1/\partial y = 0$, so $\vec{F}$ could be a gradient. ($f(x, y) = \frac{1}{2}x^2$ is a potential function.)

4. $\partial G_2/\partial x = -2y$, $\partial G_1/\partial y = -2y$, so $\vec{G}$ could be a gradient. ($f(x, y) = \frac{1}{3}x^3 - xy^2$ is a potential function.)

13. (a) C_1 might be the straight line path along the x-axis; C_2 might be the top half of the circle centered at the origin, of radius half of the distance from P to Q; and C_3 might be the bottem half of the same circle.
(b) No, because it is not path-independent.

15. Let $\vec{F} = x\vec{\jmath}$, as in #13 — which we know is not path-independent — and let C be the figure-8 crossing at the origin, starting at P and going around (through Q) once to return to P.

16. $\vec{F}$ may be a constant, in which case there is no change in $\vec{F}$, but the line integral can have nonzero value. Note: By the FTC for line integrals, $\int_C \nabla f \cdot d\vec{r}$ is the total change in f, not in ∇f, along C.

17. General counterexample: C is closed, $\vec{F}$ and $\vec{G}$ are gradients of functions f and g with $f - g \neq$ constant.