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2. Suppose that this vector field $\vec{F}$ was the gradient of some f. Then the contours of f would have to perpendicular to the vectors in $\vec{F}$, so they would have to be horizontal lines. But the vectors in $\vec{F}$ are also $\vec{0}$ along the y-axis, so that would also be a contour — f would not be increasing or decreasing there — and contours cannot cross each other.

3. $f_x = 2xy$ means $f = x^2y + C_1(y)$. And $f_y = x^2 + 8y^3$ means $f = x^2y + 2y^4 + C_2(x)$. So $f = x^2y + 2y^4$ is a potential function.

4. $f_x = yze^{xyz} + z^2\cos(xz^2)$ means $f = e^{xyz} + \sin(xz^2) + C_1(y,z)$. And $f_y = xze^{xyz}$ means $f = e^{xyz} + C_2(x,z)$. And $f_z = xye^{xyz} + 2xz\cos(xz^2)$ means $f = e^{xyz} + \sin(xz^2) + C_3(x,y)$. So $f = e^{xyz} + \sin(xz^2)$ is a potential function.

5. $\partial F_2/\partial x = 0$ and $\partial F_1/\partial y = 1$, so there is no potential function.

8.
$$\operatorname{curl}\vec{F} = (-x^{-1}y^{-2} - 0)\vec{\imath} - (-x^{-2}y^{-1} - 0)\vec{\jmath} + (0-0)\vec{k} \neq \vec{0} \ ,$$
so $\vec{F}$ is not a gradient. (If this curl *had* come out equal to $\vec{0}$, then we would not have been able to conclude that $\vec{F}$ was a gradient until we were sure that there were no holes in the domain that could be encircled by curves. But the domain is everything except the planes $x = 0$ and $y = 0$, so a curve cannot encircle a "bad point" without having "bad points" on it.)

11. (a) This vector field was shown in one of the graphics used in class for Section 17.2. It had all horizontal vectors, to the right above the x-axis and to the left below. So traversing the unit circle as described would be moving against the vector field at each point (except on the x-axis, where the vectors are $\vec{0}$), so the circulation — i.e., the line integral — would be negative.
(b) Let C denote the directed circle, and let R be the region (a disc) that it encloses. Then:
$$\int_C \vec{F}\cdot d\vec{r} = \int_R (\frac{\partial 0}{\partial x} - \frac{\partial y}{\partial y})dA = \int_R (-1)dA = -\pi \ ,$$
because the last integral is the negative of the area of the region.

12. If C is the closed curve and R is the region it encloses, then by Green's Theorem,
$$\int_C \vec{F}\cdot d\vec{r} = \int_R (\frac{\partial x}{\partial x} - \frac{\partial 0}{\partial y})dA = \int_R (1-0)dA = \int_R dA = \text{Area}(R) \ .$$