Math 312 Unit 6 Spring 2017

Due in class: Monday April 3, 2017

1. Consider a random walk (this of a drunkard coming home from a bar) on a one-dimensional line with 5 locations. At one end is the drunkard's house. At the other the police station. The drunkard is kicked out of the bar in the middle location and will stay at home or in the police station if those states are reached. At any given interval, the drunkard moves up or down the street with 40% chance of moving toward the police station and 60% chance of moving toward the house.

   a) Define variables that describe the state of this system.

   b) Is this process a linear stationary Markov chain?

   c) Show whether this transition process is regular? Identify any absorbing states?

   d) Write the transition matrix for this process with any absorbing states listed as the first rows and other states in the following rows. Identify which rows and columns correspond to which states.

   e) Derive the expected number of time intervals the drunkard is "on the street", that is between the time that the bar closes and the time the drunkard stops for the night.

   f) Derive the probability of the drunkard ending up at the police station.

2. An absorbing linear stationary Markov chain model has two absorbing states and three transient states. The submatrices for the transition matrix are:

$$R = \begin{bmatrix} 0.051 & 0 & 0.500 \\ 0 & 0.082 & 0.500 \end{bmatrix}$$

   and

$$Q = \begin{bmatrix} 0.588 & 0.490 & 0 \\ 0.360 & 0.429 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

   The fundamental matrix is:

$$N = (I - Q)^{-1} = \begin{bmatrix} 9.714 & 8.327 & 0 \\ 6.125 & 7.000 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   and transition to absorbing submatrix

$$RN = \begin{bmatrix} 0.500 & 0.429 & 0.500 \\ 0.500 & 0.571 & 0.500 \end{bmatrix}$$

   a) What is the chance of ending up in the second absorbing state if we start in the first transient state?

   b) How many times would we expect to be in the second transient state if we start in the first transient state?

   c) How many timesteps should we expect to have to wait until absorption occurs if we start in the second transient state?

3. Suppose you come up with a very good short term trading scheme for stocks that allows you to gain money on average. A typical scenario is that you wait for a certain pattern in the stock chart and then invest \$1000. A really good pattern might give you a \$1100 payback 51% of the time and a \$900 payback 49% of the time. Suppose you have \$10,000 to invest at the start and you keep investing over and over again.

   a) Introduce variables that describe the state of the system.

   b) Is this transition process regular? Are there any absorbing states? Describe your answers mathematically and then interpret what they mean in terms of the setting we are describing.

   c) We can't say what will happen in the long run for this probability distribution based on what we have discussed in class. Why not?

4. In marketing, you can quantify how fashion changes over time. For example, the color of coats worn in one season depends on the colors that were popular the previous season. Suppose that black, white and tan are the only colors considered and the proportion of coats sold in each is b, w, and t respectively. Suppose that people who buy black coats this year will switch to white next year at a percentage rate that is proportional to how many white coats were sold this year. Let the proportionality constant be B. For simplicity, we'll ignore all other switches so that the rest of the black coats stay black, tan stay tan, and white stay white.

   a) Write out the formula for white proportion of coats at the new time w_{k+1} as a function of the proportions at the old time b_k, w_k, t_k.

   b) Explain why our methods don't apply in this case.

   c) Could you simulate this process on a computer to get the long term probability distribution? What method is that?