Math 323 — Practice Exam II, part 1

Prove or give a counterexample in each case: For a continuous function from a subset of \mathbf{R} into \mathbf{R} the \mathcal{Q} of a \mathcal{P} set is a \mathcal{P} set (in the domain of the function).

$\mathcal{P}\setminus\mathcal{Q}$	image	inverse image
open		T: equivalent to "continuous"
closed		T: equivalent to "continuous" 2
bounded	F: 3	
compact		
connected 1	T: 4	

Notes:

- 1 For a subset A of \mathbf{R} , the following are equivalent:
 - (a) A is connected.
 - (b) A is an interval (of some kind: bounded or unbounded; open, closed or half-open).
 - (c) For all $r, s, t \in \mathbf{R}$ with r < s < t and $r, t \in A$, we have $s \in A$.

2 For any function $f : A \to B$ and any subset C of B, we have $A \setminus f^{-1}(C) = f^{-1}(B \setminus C)$; so "inverse images respect complements". Verify this equality of sets, and conclude that the conditions "the inverse image of every open set is open" and "the inverse image of every closed set is closed" are equivalent.

3 This one is false, and it is the only one where you can't get a function f coming from all of **R**.

4 This is the Intermediate Value Theorem — we'll do it in class.